MATHEMATICAL MODELLING OF PHYSIOLOGICAL FLUID SYSTEMS : EFFECTS OF PERISTALSIS AND MULTIPLE STENOSES

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DOCTOR OF PHILOSOPHY

By

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TO THE MEMORY OF

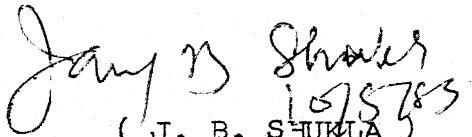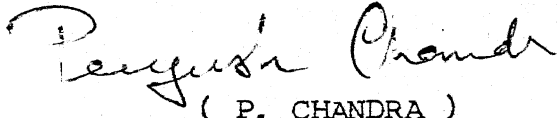MY FATHER

(LATE) SHRI RAMESHWAR PRASAD SHARMA

CERTIFICATE

This is to certify that the matter embodied in the thesis entitled ''MATHEMATICAL MODELLING OF PHYSIOLOGICAL FLUID SYSTEMS : EFFECTS OF PERISTALSIS AND MULTIPLE STENOSES'' by Mr. Rajiv Sharma for the award of Degree of Doctor of Philosophy of the Indian Institute of Technology, Kanpur, is a record of bonafide research work carried out by him under our supervision and guidance. The results embodied in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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CONTENTS

CHAPTER I

GENERAL INTRODUCTION

1.1 INTRODUCTION

The science of physiology deals with the study of
functional behaviour of living structures. Most of the
physiological systems involve movement of some fluids in
response to mechanical forces, for example motility of
the spermatozoa and ova, propulsion of bacteria and
protozoa, blood flow through circulatory system of mammals,
etc. The characteristics of such systems are generally
studied by using the principles of fluid dynamics and
making appropriate mathematical models of the system under
consideration, after taking into account the various known
and unknown variables and parameters related with this
system. [Burton (1965), Gray (1968), Dowben (1969), Fung
et al. (1972), Rubinow (1975)].

In this thesis the following problems have been
studied.

(i)   Effects of Peristalsis on the motion of micro-organism:
       Mathematical models, to study the effects of peristal-
       sis, longitudinal motion of the wall, cilia motion
       etc. on the movement of microorganisms, have been
       obtained. The results have been applied to study
       the swimming of spermatozoa and ovum. The effect
       of viscosity variation of the physiological fluid is
       also studied in this context.

(ii) Peristaltic transport of fluid in intestines : Effects
      of gradually varying cross-section and multiple
      constriction in a tube have been studied on the
      peristaltic transport of fluid.

(iii) Blood flow in arteries : Effects of non-uniform
      geometry and multiple stenosis on the blood flow
      through arteries have been investigated by considering
      the non-Newtonian nature of the blood and its
      consistency variation across the lumen.

1.2 PERISTALSIS

Peristalsis is the mechanism by which fluid is
transported through a distensible tube when contraction or
expansion waves propagate along its length. Peristalsis
appears to be the mechanism for fluid transport in many
physiological situations such as transport of urine through
ureter, food mixing and chyme movement in intestines, transport
in bile duct, etc. The study of peristaltic transport
of fluid is based on the well known principles of fluid
mechanics involving interaction of fluid motion in tubes
with flexible boundaries . In such investigations an approp-
riate mathematical model of the physiological system is made
by keeping in view the nature of the biofluid (i.e. its
Newtonian or non-Newtonian character, its behaviour as a two
phase mixture, its viscosity or consistency variation across

the lumen, etc.), the nature of the tube (i.e. its elasticity, presence of cilia on the inner wall, nonuniformity of its cross-section due to constriction, etc.) and other processes (heat and mass transfer, blending and mixing, steady and unsteady behaviour, etc.) involved.

Several investigators have contributed to the study of peristaltic action in both mechanical and physiological situations e.g. Burns and Parkes (1967), Hanin (1968), Barton and Raynor (1968), Fung and Yih (1968), Shapiro et al. (1969), Yin and Fung (1969), Chow (1970), Zien and Ostrach (1970), Li (1970), Lykoudis and Roos (1970), Weinberg et al. (1971), Mittra (1971), Jaffrin and Shapiro (1971), Tong and Vawter (1972), Brown, et al. (1977), Wilson, et al. (1979). An elaborate review of the earlier literature regarding peristalsis has been provided by Jaffrin and Shapiro (1971). In particular, Burns and Parkes (1967) used perturbation techniques to study the peristaltic motion through a channel and a tube. Barton and Raynor (1968) studied the peristaltic motion in a circular tube by using long and short wave length approximations. The fluid mechanics of the Ureter has been studied by Lykoudis and Roos (1970) and Boyarsky and Labay (1972). The interaction of Poiseuille flow on the peristaltic motion has been studied by Mittra and Prasad (1974). Gupta

and Seshadri (1976) have investigated the peristaltic
pumping in a non-uniform tube. Peristaltic motion of a
solid fluid mixture has been studied by Kaimal (1977).

In these studies, the effect of viscosity variation
of the fluid has not been taken into account, though
there have been suggestions that peristaltic mechanism
may be involved in vasomotion of small blood vessels
[Fung and Yih (1968)] ; in ducts efferentes of the male
reproductive tract [Lardner and Shack (1972)] ; in
transport of spermatozoa in cervical canal [Shukla et al.
(1978)] and in intestines- all being cases where viscosity
of the fluid near the wall is different than its viscosity
in the centre of the duct. Keeping this in view, Shukla,
et al. (1980d) studied the effects of peripheral layer
viscosity on the peristaltic transport of biofluid and
compared the results with experimental data obtained for
flow rates in intestine and ductus efferentes of the male
reproductive tract.

Though it is known by experiments of Patel et al. (1973)
on human feces that it behaves rheologically as non-Newtonian
power law fluid, little attention has been paid to the study
of peristaltic transport of such fluids [Raju, et al. (1972)].
Picologlou et al. (1973) have noted that peristalsis in
colon, though rare, is a major propulsive mechanism and

presented an analysis to study the movement of human feces
by considering it as a non-Newtonian power-law fluid.
Radhakrishnamcharya (1982) has also analysed the peristaltic
transport of a power law fluid. In view of the power-law
non-Newtonian behaviour of the chyme and the presence of
peripheral mucus layer in intestines, Shukla, et al. (1982)
have studied the peristaltic transport of a power law
fluid whose consistency varies along the radial direction
and predicted the effects of peripheral layer consistency
and the pseudoplastic nature of the biofluid on its flow
characteristics.

The effects of peristalsis with particular reference
to spermatic fluid transport in vas deferens has been
studied [Gupta and Seshadri (1976) and Srivastava et al.(1983)]
by considering   the duct with cross-section varying
linearly along the length in the stationary frame.

It is evident, thus, that peristalsis plays an important
role in the transport of physiological fluids. It is,
therefore, interesting to study the effects of peristalsis
on bio-systems like swimming of micro-organisms such as
spermatozoa and ovum, and on transport of contents of a tube
with varying cross-section such as intestines with constrict-
ion due to polyp. Keeping this in view, various models are
presented in chapters II-V related to these aspects.

1.3 EFFECTS OF PERISTALSIS ON THE MOTION OF MICRO-ORGANISMS

Micro-organisms like spermatozoa, nematodes, bacteria etc. are known to swim through a fluid by sending lateral waves of displacement along their tails or flagella. In order for a micro-organism to propel itself with a constant velocity, it should move in such a way that the forces due to pressure and viscous stresses, exerted by the surrounding fluid on its body, must be balanced. It is also to be noted that for swimming of these minute bodies, the Reynolds number is of order 10^{-3} or less [Taylor (1951)] so that the propulsion is due predominantly to viscous forces. In such low Reynolds number problems the equations governing the fluid motion are the linearized Navier-Stokes equations, (Stokes equation) [Shen et al. (1975)].

As the model proposed in this thesis regarding the motion of micro-organisms have been particularly applied to understand the transport of spermatozoa and ovum in human genital organs, it would be of some interest to discuss the structures of spermatozoon and ovum and related physiology, before reviewing the relevent current literature. The proper understanding of these phenomena is necessary as it may provide an insight into the process of fertility and may suggest a biochemical or biomechanical means to control the human population.

Structure of Spermatozoon

The spermatozoon is a specialised cell which contains only the half of the usual 46 chromosomes present in other cell of human body. Since the nucleus of the female germ cell 'ovum' also contain 23 chromosomes, the eventual combination of spermatozoon and ovum reestablishes a normal cells of 46 chromosomes which is capable of growing by division. The total length of spermatozoon is 60 μm ($1 \mu m = 10^{-6}$ meter) approximately equal to the diameter of the nucleus of ovum. The structure of a spermatozoon is shown in fig. 1.1. Its body can be divided into head and tail. On the basis of slight differences in the thickness the tail is further divided into neck, middle piece, main piece and end piece [Warwick and Williams (1973), Vander et al. (1977), Chatterjee (1980)].

The head is oval on surface view, and measures about 4-5 μm in length and 2.5-3.5 μm in diameter. It is a condensed nucleus, containing about 40 o/o DNA bearing the sperm's genetic information, and is covered by a cap called acrosome. This cap contains several lytic enzymes to enable the sperm to pierce the zona pelluicida of the ovum.

The neck connects head with the middle piece and its axial filaments consist of three bundles of fibrils being connected with the three basal granules in the anterior centriole.

nucleus

nuclear membrane

head

anterior
knob

cell membrane

neck

basal body

posterior knob

axial fibres

mitochondrial sheath

middle
piece

ring centriole

main piece of
tail

capsule

fibrous sheath

axial filaments

terminal end piece of tail
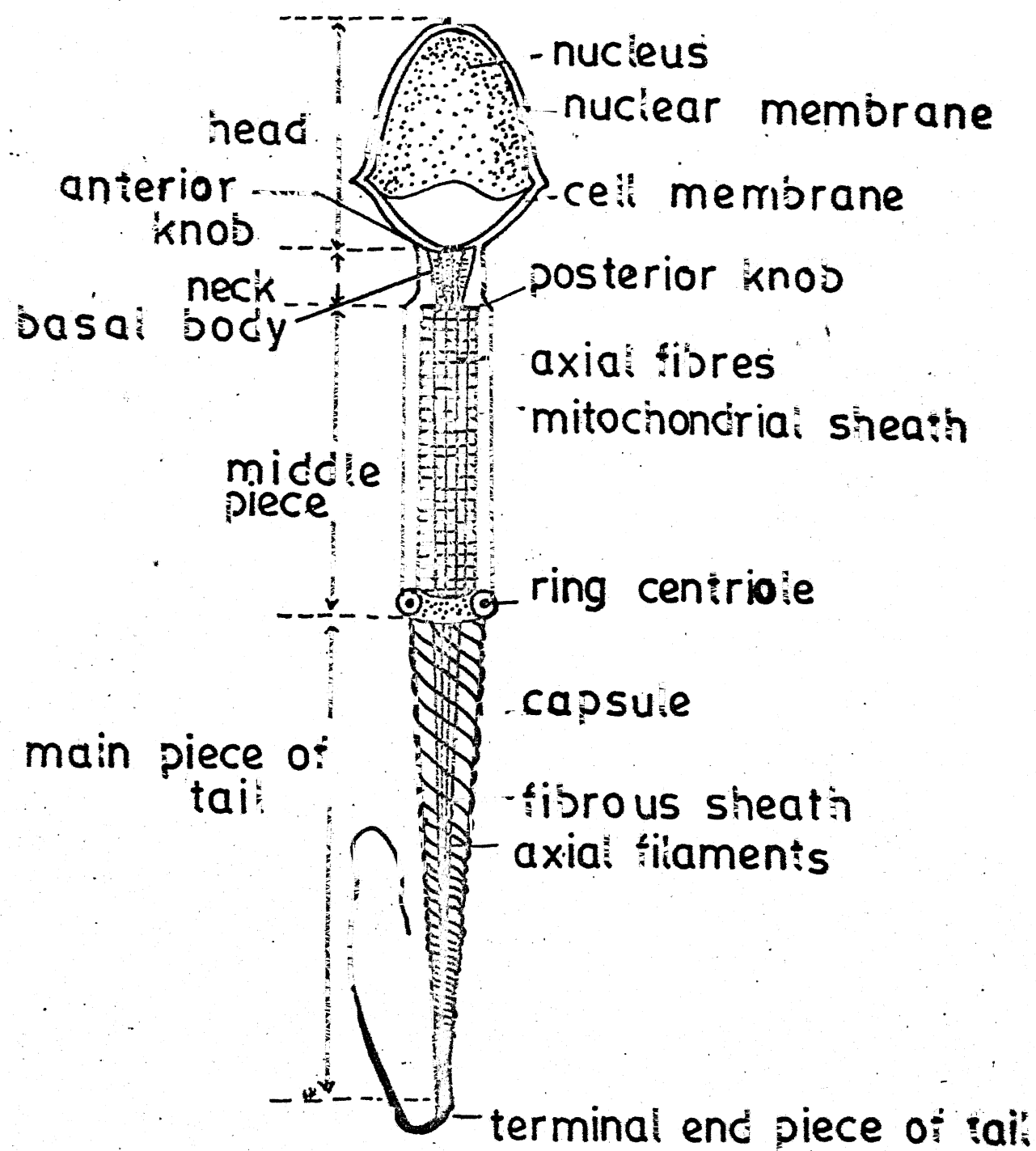
Fig. 1.1 Front view of Spermatazoon

The middle piece is the engine room of the spermatozoon which controls the whiplike action of the tail. It is cylindrical in shape, 5-7 μm long and 1 μm. wide at base.

The main piece of the tail is the largest part of the spermatazoon. It is 45 μm long and about 0.5 μm thick at base, gradually tapering towards the end piece. It comprises a group of actomysin-like contractile filaments, arranged in nine-plus-two structures i.e. nine pairs of fibrils form a regular array in an outer ring of cross-section and two single fibrils are in the center. Contraction of these filaments produces a whip-like movement of the tail capable of propelling the sperm.

The end piece of the tail or Flagellum is the terminal portion the axial filament and is about 5 μm long. Its cross-section is identical to that of a cilium.

MALE REPRODUCTIVE PHYSIOLOGY

The male reproductive system essentially consists of spermatozoa (described above), testes and the male genital tract . The testis whose cross section is shown in fig.1.2, is primarily composed of many highly coiled seminiferous tubules, whose combined length is approximately 250 meters. These tubules are lined with number of cells which are continuously regenerating themselves and thus providing a continuous source of spermatids which are
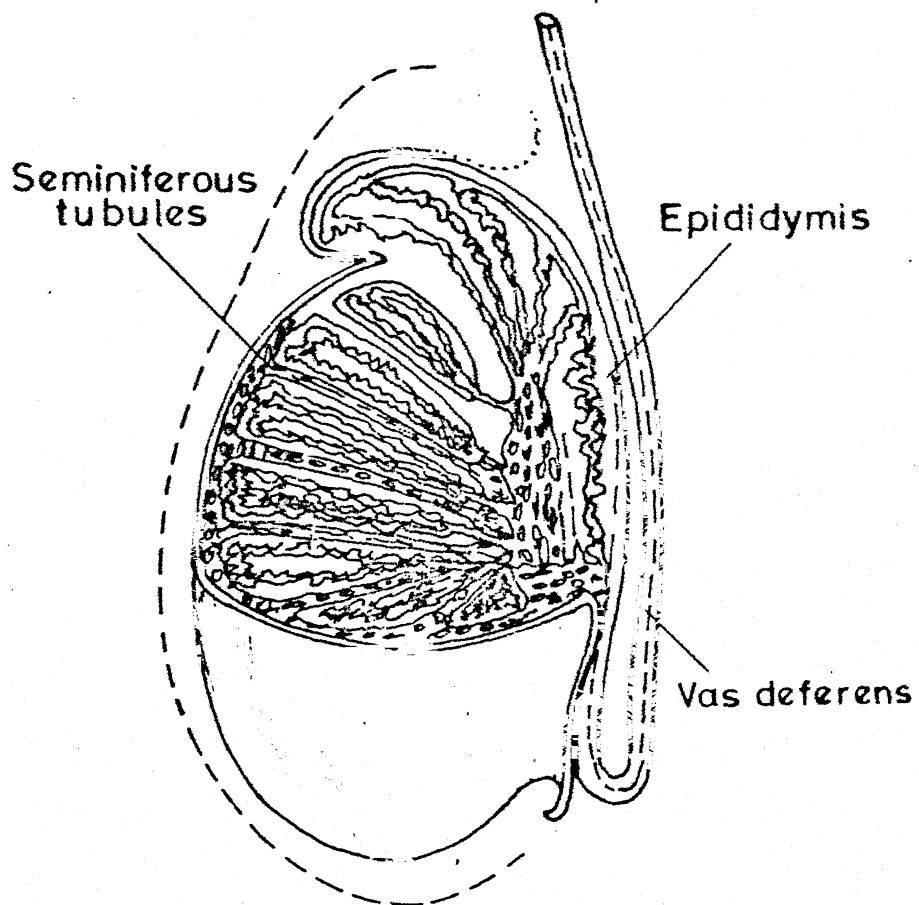
FIG. 1·2 DIAGRAMMATIC CROSS SECTION OF
A HUMAN TESTIS.

ultimately matured as spermatozoa. The other component
of testis consists of interstitial cells which secrete
the male sex harmone 'testosterone', necessary for sperm
production.

The anatomical organization of the male genital
duct system is shown in Figure 1.3. From the seminifenous
tubules, the sperm pass through a network of interconnected
highly coiled ducts joining to form 'epididymis' which leads
to large thick walled 'vas deferens'. The sperm themselves
are non-motile at this stage and their transport is achieved
by means of peristaltic action of contractile cells in the
duct walls. The epididymis and initial portion of vas
deferens store spermatozoa, prior to ejaculation. At
ejaculation the spermatozoa are expelled, by strong contractions
of smooth muscles lining these duct walls, to be deposited
at the mouth of cervix in female genital tract.

STRUCTURE OF OVUM:

The structure of an ovum is shown in fig. 1.4. It
consists of a nucleus (germinal vesicle) which is approximately
equal to 60 μm in diameter (the length of a spermatazoon is
55-60 μm). This nucleus alongwith the ooplasm centrosome
is suspended into a sac filled with cytoplasm. This sac is
surrounded by the zona pellucida and a region ''perivitelline
space'' is left in between.
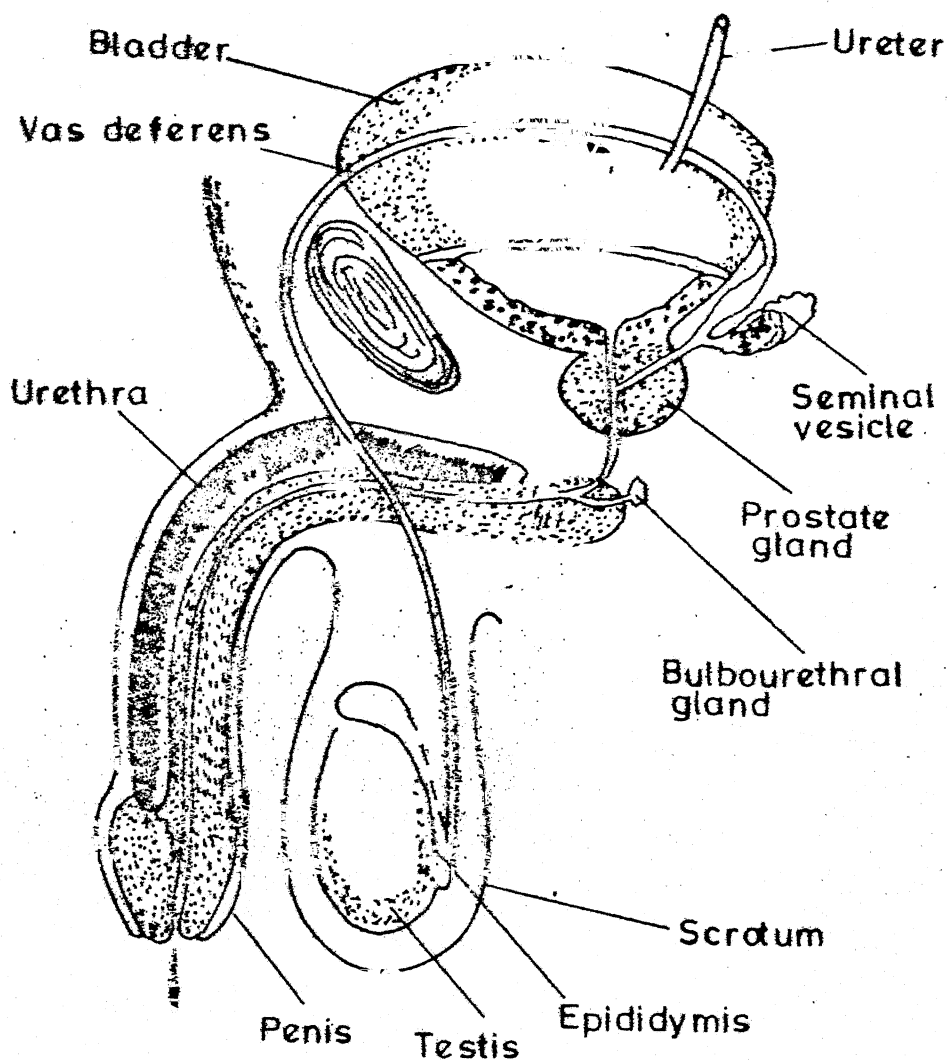
FIG. 1.3   ANATOMICAL ORGANIZATION OF THE MALE
REPRODUCTIVE TRACT
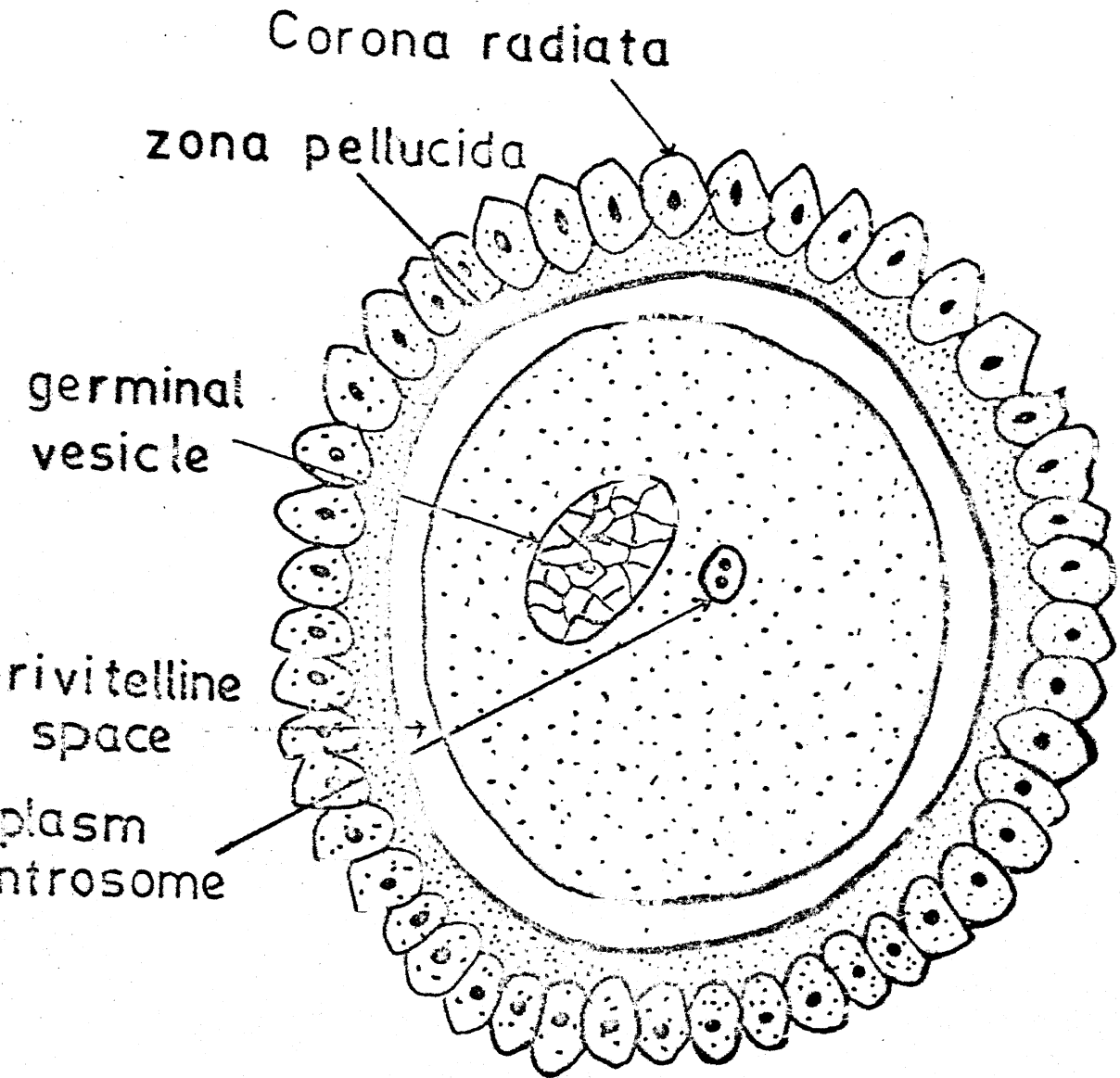
FIG. 1.4 : OVUM SURROUNDED BY DISCUS CELLS

FEMALE REPRODUCTIVE PHYSIOLOGY

The female reproductive organs consist of :

(i)   Ovaries ; which are the female counterpart of the
       male testis. It produces ova and female sex harmones
       like estrogen and progesterone.

(ii)   Oviduct or fallopian tube : It is a trumpet shaped
       duct and is 10 cm. long. The process of fertilization
       takes place in this tube. It is known to have fingerlike
       projections at the ovarian end which sweep over the
       ovary during ovulation.

(iii) Uterus : It is a pear shaped organ. It is on
       average 7.5 cm long.   The length of the uterine
       cavity measures 6 cm. from the external os to the
       fundus. Its function is to provide nutrition to the
       fertilized ovum which is now known as blastocyte
       and is grown as embrayo.

       Other sexual organs consist of vagina which is
       approximately 7.5 cm. long and the cervix-which
       connects vagina with the uterus. (see fig. 1.5).

OVULATION :

       Each ovary contains a vast number of descrete cell
clustures known as primordial follicles. Each follicle is
composed of one ovum surrounded by a single layer of flattened
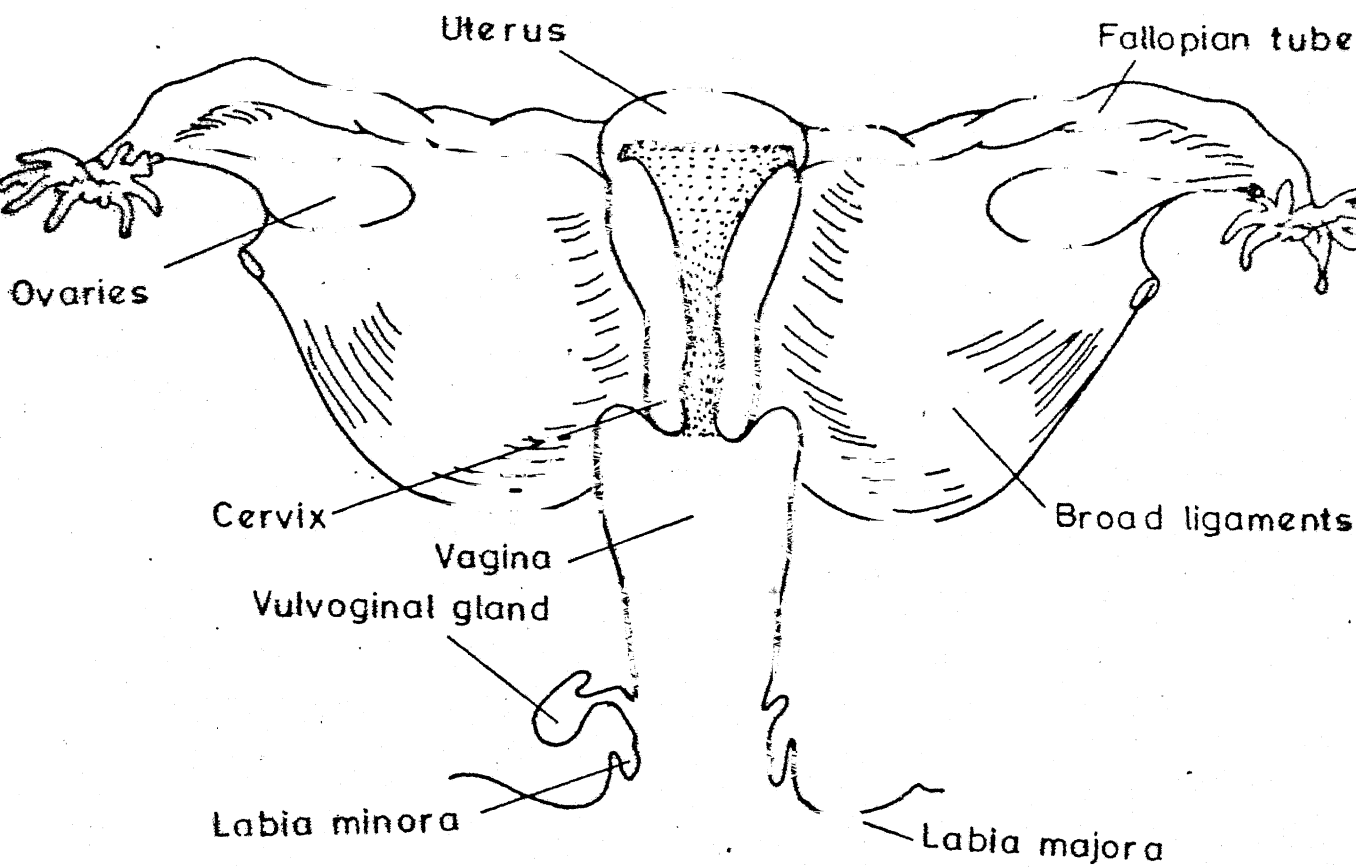
FIG. 1.5 CROSS SECTION OF THE FEMALE REPRODUCTIVE SYSTEM

cells. At birth, normal human ovaries contain an estimated
number of 400,000 such follicles and no new follicle
is made afterwards. Some 400 of these reach full maturity
during active sexual life of a woman and all other degenerate.
The development of a follicle is characterized by an
increase in size of the ovum and a proliferation of the
surrounding follicle cells. The ovum becomes seperated
from the follicle cells by a thick membrane called zona
pellucida. The ovum is surrounded by a fluid-filled space
antrum which starts increasing enormously and thus pushing
the ovum, surrounded by its zona pellucida, towards the
thin membrane of the follicle . The thin wall of the
follicle balloons out on the surface of ovary and the
ovulation occurs when this membrane ruptures. Due to
the pressure exerted by the antral fluid, the ovum alongwith
the zona pellucida and tightly adhering follicle cells
(known as corona radiata) is extruded into the ovarian
end of the oviduct. The process of ovulation takes
approximately 2 weeks after the menstrual flow of the
previous menstrual cycle.

TRANSPORT OF OVUM AND SPERMATOZOA :

At the time of ovulation the ovum is extruded onto
the surface of the ovary near the ovarian end of the
oviduct and is sucked in the oviduct because of the

sweeping motion of the fingerlike projections at this
end of oviduct, caused by the contraction and relaxation
of the smooth muscles surrounding them. The epithelium of
the oviduct is lined with cilia-hairlike projections
of the walls, beating in waves towards the interior of the
duct. This motion of cilia felicitate the motion of the ovum.
It is speculated that it takes several days for ovum to
reach the uterus, hence the process of fertilization must
occur in the oviduct itself.

The spermatozoa are deposited , during the coitus,
at the mouth of the cervix and are known to travel at an
average speed of 1-3 mm./minute. The dimension of the
female genital tract, traversed by the spermatozoa are
given below

Length of vagina   7.5 cm.

Length of the uterine cavity   6 cm.

(from external os to the

fundus)

Length of the uterus   7.5 cm.

Length of the oviduct   10 cm.

The transport of the sperm to the site of fertilization
(in the oviduct) is so rapid that the first sperm arrives
within 45 minutes. This is far too rapid to be accounted
for by the sperm's own motility. It is suggested that

the movement produced by the wave motion along the sperm's
tail is probably essential only for the final stages of
approach and penetration of the ovum. The act of coitus
provide some impetus because of the ejaculated fluid pressure
and the pumping action of the penis. The actual mechanism
of spermatozoa transport is still under speculation.

The study of swimming of micro-organisms was initiated
by Taylor (1951) who modelled it as motion of a fluid
near a sheet consisting of spermatozoa, down which waves
of lateral displacement are propagated. Hancock (1953)
studied propulsion of a thin circular filament through a
cyclindrical tube, for several wave amplitudes and radii
of of tube. Further studies were conducted for small wave
amplitudes and long wavelength approximation by Gray and
Hancock (1955), Reynolds (1965), Shack and Lardner (1974),
Shack et al. (1974), Shen et al. (1975). Attempts were also
made to explain the motion of spermatozoa in the female
genital tract by considering the dynamical interaction of
the wall [Smelser et al. (1973), Shukla et al. (1978)]. Blake
et al (1983) also presented a theoretical model of ovum-
transport in oviduct incorporating transport mechanisms
due to ciliary and muscular activity by adding a force
distribution term in the equation of motion. The motion
due to ciliary and muscular activity can also be represented

by a combination of transverse and longitudinal waves
along the wall. It is also suggested that peristalsis
plays an important role in transport of spermatic fluid
through male genital tract [Guha et al. (1975), Gupta
and Seshadri (1976) and Srivastava et al. (1983)].

In above mentioned studies the interaction of the
wall motion on the swimming of micro-organism has not
been considered. Therefore in this thesis mathematical
models are presented to study the motion of self-propelling
micro-organism when the wall of the duct is undergoing
peristaltic and longitudinal motion. It may be pointed out
that the peristaltic motion of the wall could arise due
to contraction and relaxation of the smooth muscles of
the uterus and the oviduct while the longitudinal motion
could be due to cilia on the walls of the oviduct. The
effects of peristalsis/longitudinal motility of the wall
have been investigated by many researchers in other physical/
biological situations. [Barton and Raynor (1968), Fung
and Yih (1968), Shapiro et al. (1969), Zien and Ostrach
(1970), Shack and Lardner (1972), Macagno et al. (1975),
Shukla et al. (1980d),Blake et al. (1983)] and here we
investigate these effects to study the motion of micro-
organisms. As one of the aims of proposed study in this
thesis is to apply the analysis to fertility control by

reducing the speed of micro-organism, the direction
of the peristaltic waves is taken to be opposite to
that of its motion while studying the movement of
spermatozoa and ovum.

EFFECT OF VISCOSITITY OF THE PHYSIOLOGICAL FLUID

Another important factor affecting the sperm-
transport and ovum - movement is the viscosity variation
of the bio-fluid across the cross-section of the duct.
It is observed that the viscosity of the cervical mucus
is higher near the walls in the mid - period of the
menstrual cycle due to concentration of the micelles
and this aspect should be taken into account for studying
the motion of the spermatozoa. A detailed account of
this topic has been given by Scott Blair (1974), Cliff
et al. (1950) studied the data related to humans and
observed the consistency of the mucus at the time of
ovulation to be minimum . Chemically this change is
due to the effects of the harmones on the mucus composition.
The physical aspects of this change have been studied
for swimming of spermatozoa in semen and in cervical
mucus, Lamar et al. (1940) experimentally showed that
the penetration of the cervical mucus by the spermatozoa
was maximal for the mucus collected at about the 14th
day of the cycle which coincides approximately with the

time of ovulation in most of the women. This behaviour
of the cervical mucus was suggested to be utilised as a
means of fertility control in humans [James and Marriott (1983)].

The cervical mucus is a suspension of macromolecules
in a waterlike liquid (Odeblad (1959,1962)]. It is observed
that in luteal phase the mucus resembles like a close
mesh, with macromolecules spreading all over the cervical
canal with a spacing of 0.03 μm between them. However in
the mid cycle period the molecules group together to form
micelles which align themselves along the walls of the
cervical canal [Odeblad (1968), Elstein (1971), Davajan
et al. (1971)]. Because of this alignment of micelles
along the wall of the duct, a variation in the viscosity
of the mucus is introduced and its penetration by the
sperm becomes possible near the axis of the duct.

To summarize it may be noted that the spermatozoa
swim by sending lateral waves along its tail while the
ovum does not have its own motility. Further their motions
are affected by the contraction and relaxation of the smooth
muscles of uterus and oviduct. It is also pointed out
that the female reproductive organs, especially the fallopian
tube are lined with hair-like projections called 'cilia',
whose motion affects the movement of the ovum. This motion
of cillia is generally represented by a combination of

transverse and longitudinal waves. Another important
factor affecting transport of these micro-organisms
is the viscosity variation of the physiological fluid
across the duct. In view of these, new models are proposed
in this thesis, to study, in particular, the effect
of following factors on the movement of spermatozoa in
cervical canal and ovum in fallopian tube. (Chapters II, III,
IV):

    (i) Peristaltic motion of the walls

    (ii) Longitudinal motion of the walls

    (iii) Viscosity variation of the physiological fluid.

1.4 INTERACTION OF NON-UNIFORM GEOMETRY CONSTRICTIONS WITH THE PERISTALTIC MOTION

The second problem studied in this thesis is
associated with the transport of feces in intestines
due to peristalsis. It is observed that cross-section of
the intestinal duct gradually varies along its length.
Further there may be constrictions due to polyp etc.
in the lumen of the intestine. The dynamical interaction
between the passage of the progressive peristaltic waves
and an abnormal growth in the lumen of the transporting
duct may be very complex and its study may lead to a better
understanding of such pathological situations.

As stated earlier (in section 1.2), several authors
have contributed to the study of peristaltic transport
in physiological as well as mechanical situations [Burns
and Parkes (1967) , Fung and Yih (1968), Barton and
Raynor (1968), Shapiro et al. (1969), Li (1970), Zien and
Ostrach (1970), Raju and Devanathan (1972), Tong and
Vawter (1973), Manton (1975), Kaimal (1978), Shukla et al.(1978),
R.K.Acharya(1982) , Shukla and Gupta (1982)].

It must be observed that in these studies the solutions
have been obtained in the wave frame, where the flux is
steady because of the uniform geometry of the transporting
ducts. However, it is noted that cross section of various
ducts in human body gradually varies along their length and
therefore a steady flow through them can not be obtained
even in the wave frame. Guha et al. (1975) studied the
effect of peristalsis on the flow through a duct of linearly
varying cross - section with particular reference to spermatic
fluid transport in vas deferens. Later Gupta and Seshadri
(1976) and Srivastava et al. (1983) also studied this problem
and obtained the solution in the stationary frame. In these
studies the value of pressure difference across a wavelength
has been calculated for a given rate of flow and periodic
nature of the pressure difference has been shown.

It may also be pointed out here that the peristaltic

transport of fluid in constricted tubes has not been
studied so far, though such situations may arise due to
polyp, etc. in the lumen of the intestine. A model is,
therefore, developed in chapter V, to study the effect
of non-uniform geometry and multiple constriction on
peristaltic transport of fluids through a circular duct,
with reference to intestines. Following Gupta and
Seshadri (1976), solutions are obtained in the fixed
laboratory frame using long wavelength approximation.

1.5 FLOW OF NON-NEWTONIAN FLUID THROUGH A TUBE WITH MULTIPLE
STENOSES

Another important problem dealt in the thesis is
related to blood flow in arteries with multiple stenoses.
Stenosis is the term used for the abnormal and unnatural
growth in the lumen of the arteries carrying blood from
heart to various parts of the body. It is caused mainly
due to intravascular atherosclerotic plaques which develop
at the arterial wall, thus occluding the lumen of the artery
partly or fully. It has been suggested that hydrodynamic
factors such as wall stress, pressure distribution and
seperation phenomenon play an important role in the development
and progression of stenosis [Rodkiwitcz (1977) and Stehbens
(1982)]. Hence many theoretical and experimental studies
of blood flow in stenotic region under different conditions
have been conducted, [Young (1968), Caro et al. (1971),

Nerem (1974), Deshpande et.al. (1976), Mc Donald (1979),
Shukla et.al. (1980a), Padmanabhan and Devanathan (1981)
and Talukder et al. (1976)]. Most of these analytical
studies have investigated the flow characteristics of
blood in arteries with mild/non-critical stenosis by
considering blood as a Newtonian Fluid the survey for
which can be found in Young (1979). However, it may
be pointed out that blood being a suspension of cells
in plasma, has been found to behave as a non-Newtonian
fluid at low shears rates in tubes of small diameters.
In view of this some investigators have considered non-
Newtonian behaviour of blood [Shukla et al. (1980b and c),
Williams and Javadpour (1980), Devanathan and
Parvathamma (1983)]. Shukla et al. (1980b) studied the
flow characteristics for mild stenosis by considering
a non-Newtonian power law fluid model for blood. They
have also considered the effect of peripheral layer on
the flow of blood to take into account the clustering of
cells in the central zone [Shukla et al. (1980b)].

In above mentioned studies only the effect of
single stenosis has been considered and the tube was
taken to be of uniform cross-section. However it may
be noted that many of the blood vessels change their
cross-section slowly along their lengths [Schneck et al.
(1975)] and may have multiple stenoses at junctions, bends

etc. Talukder et al. (1976) have conducted experiments
in the case of blood flow in artery with multiple stenoses
and pointed out that the pressure drop increases linearly
with numbers of stenoses. Keeping this in view, in
chapter VI a model has been proposed to study the effect
of non-Newtonian power law behaviour of blood and its
consistency variation near the wall on the flow through
the artery with varying cross-section having two stenoses,
one in the uniform region and the other located at the
junction.

1.6 SUMMARY

The thesis is divided into six chapters with
chapters III and IV being further divided into two
sub-sections.

In chapter I, physiological details and brief survey
of the problems and summary of the results are given.

In chapter II, a mathematical model is presented
to study the motion of the spermatozoa in cervical
canal by considering the transverse waves along its tail
and the transverse and longitudinal motions of the cervical
wall. The condition of equilibrium of forces on its body
has been considered. As an attempt to control fertility
by reducing the speed of sperm, the transverse waves have
been considered in the direction opposite to the motion

of the spermatozoa. It has been shown that by having
appropriate transverse wave motion and longitudinal velocity
the sperm may not be able to move towards oviduct even
if it could continue to have its own propelling velocity.

In chapter III the motion of micro-organisms of finite
thickness, in a fluid flowing through a channel and a tube
is studied under peristaltic and longitudinal motion of the
walls. The symmetric motion of the transverse waves along
the surface of the micro-organism is also taken into account.
The condition of equilibrium of forces on the micro-organism
is used in making the mathematical model. It has been shown
that even in the presence of its own motion, it may not be
able to move due to presence of transverse and longitudinal
motion of the wall, when these are applied appropriately.
This model is used to study the motion of ovum in the fallopian
tube as a particular case. As the motion of ovum is enhanced
due to the presence of cilia motion, it has been shown that
for fertility control the peristaltic and longitudinal motions
should be as minimum as possible, i.e. cilia should become
either immobile or reverse their motion.

In chapter IV the same problem has been studied where
the effect of the variation of the viscosity due to menstrual
cycle is investigated. Apart from the above mentioned results
in chapter III, it has been shown that the velocity of the

micro-organism decreases as the viscosity of the peripheral
layer increases.

A mathematical model for peristaltic transport of
a Newtonian fluid through a tube with non-uniform cross-
section and multiple constrictions is proposed in chapter V.
The shear stress at the constriction is observed to rise
considerably because of peristalsis and convergence of the
tube.

Finally in chapter VI a model of blood flow as power
law fluid, through an artery having multiple stenoses is
studied. Effects of peripheral layer and the varying cross-
section have been investigated. It is found that the shear
stress and the resistance to flow increase due to increase
in consistency of peripheral layer, convergence of the tube
and the presence of stenoses.

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CHAPTER II

EFFECT OF PERISTALTIC AND LONGITUDINAL WAVE MOTION OF
THE CHANNEL WALL ON MOVEMENT OF MICRO-ORGANISMS :
APPLICATION TO SPERMATOZOA TRANSPORT

2.1 INTRODUCTION:

In recent years many investigators have studied the motion
of self-propelling micro-organisms swimming in a fluid [Taylor
(1951), Hancock (1953), Gray and Hancock (1955). Reynolds (1965),
Shack and Lardner (1974), Shack et al. (1974), Smelser et al.
(1974), Lighthill (1974), (1975), (1976), Rubinow (1976)].
In particular, Taylor (1951) studied the propulsion of spermat-
ozoa by modelling it as a two-dimensional, infinite extensible
sheet of zero thickness with a sinusoidal wave travelling down
its length; and Hancock (1953) considered the self propulsion
of a thin circular filament and calculated its propulsive
velocity. Later Reynolds (1965) studied the swimming of micro-
organism in a fluid for small Reynolds number and small wave
amplitudes. Shack and Lardner (1974) also studied the motion
of a micro-organism represented by a thin sheet, in a channel
under long wavelength approximation and have shown that its
speed increases as the amplitude of the propelling wave increases.

In above mentioned studies the interaction of the wall
motion on the swimming micro-organism has not been considered.
Therefore, in this chapter, a mathematical model is presented
to study the motion of self-propelling micro-organism when the

wall of the channel is undergoing peristaltic and longitudinal wave motion. It may be pointed out here that the peristaltic motion of the wall could arise due to contraction and relaxation of the smooth muscles of the uterus and the oviduct while the longitudinal motion could be due to the presence of 'cilia', the hair-like projections on the walls of the oviduct. These effects have been investigated by many researchers in different situations [Barton and Raynor (1968), Fung and Yih (1968), Shapiro et al. (1969), Zien and Ostrach (1970), Shukla et al. (1980), Shack and Lardner (1972), Macagno et al. (1975)].

One of the aims of the proposed study is to apply the analysis to fertility control by reducing the speed of spermatozoa in female genital tract. It is therefore suggested that if the peristaltic waves are induced in the direction opposite to motion of the spermatozoa by some biochemical or other means, it may be possible to reduce the speed of spermatozoa to such an extent that it may not be able to reach the point of fertilization and thus avoiding fertilization.

2.2 MATHEMATICAL MODEL:

Consider the swimming of a thin propelling sheet in a Newtonian incompressible fluid flowing through a two-dimensional channel having flexible boundaries. It is assumed that the sheet, while swimming, sends down lateral waves of finite amplitude along its length. Further, peristaltic waves of finite
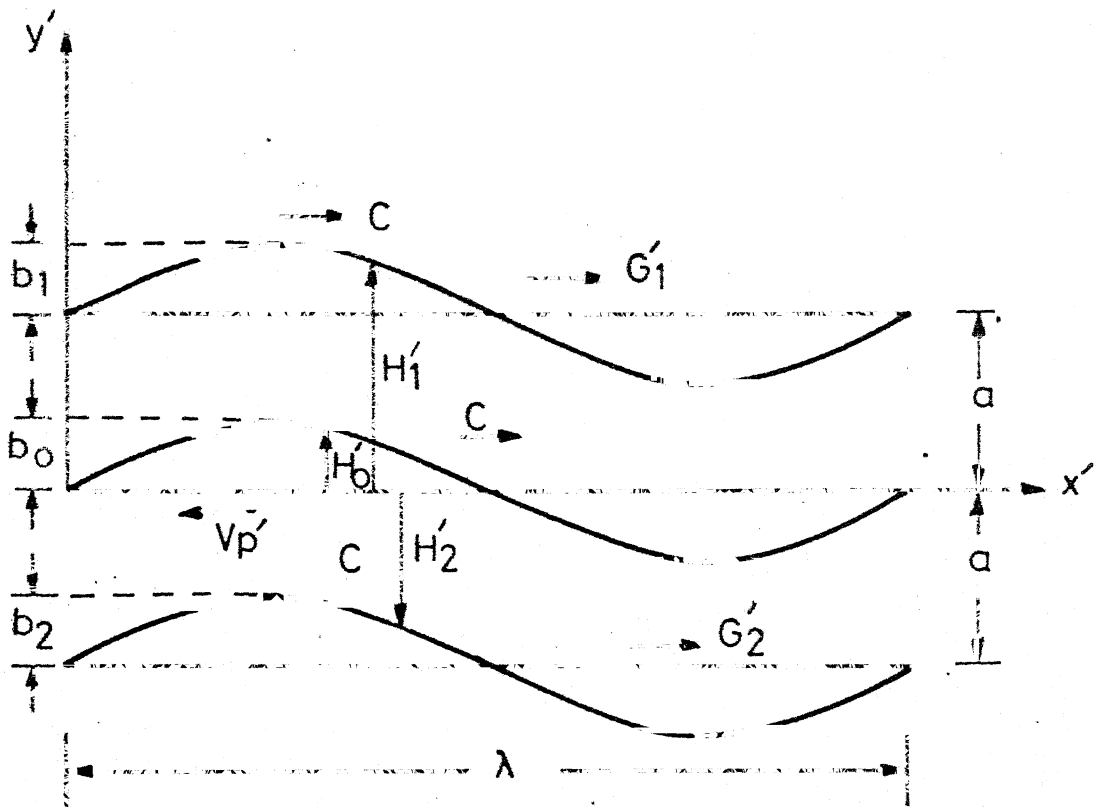
Fig. 2·1(a) Propagation of an elastic sheet
swimming through a two-
dimensional channel with peristaltic
and longitudinal motion of walls

Fig. 2·1(b) Some particular cases (i) No peristalsis
(ii) and (iii) two cases of peristalsis

amplitude are imposed along the flexible walls of the channel in the direction opposite to the motion of the sheet as shown. in Fig. 2.1. The sheet is considered to be swimming with a propulsive velocity V_p' in the negative axial direction. It is assumed that the waves travelling along the channel walls and along the sheet are in synchronization under steady state and thus have the same wave speed C (along positive axial direction) and the wavelength λ. In a fixed frame of reference (X', Y', t') the shapes of the walls and the sheet at an instant t' are given by,

$$H_1'(X', t') = a + b_1 \sin\frac{2\pi}{\lambda}(X' - Ct' + V_p't') \qquad (2.1)$$

$$H_2'(X', t') = -a + b_2 \sin\frac{2\pi}{\lambda}(X' - Ct' + V_p't') \qquad (2.2)$$

$$H_0'(X', t') = \qquad b_0 \sin\frac{2\pi}{\lambda}(X' - Ct' + V_p't') \qquad (2.3)$$

Where b_1, b_2 are the amplitudes of the peristaltic waves travelling along the upper wall and the lower wall respectively, 2a is the width of the channel and b_0 is the amplitude of the lateral wave through the sheet.

Since the Reynolds number involved in the swimming of a micro-organism is of order 10^{-3}, the governing equations for the fluid flow can be written, after neglecting the inertia terms, as follows

$$-\frac{\partial P'}{\partial X'} + \mu\left[\frac{\partial^2 U'}{\partial X'^2} + \frac{\partial^2 V'}{\partial Y'^2}\right] = 0 \qquad (2.4)$$

$$-\frac{\partial P'}{\partial Y'} + \mu\left[\frac{\partial^2 V'}{\partial X'^2} + \frac{\partial^2 V'}{\partial Y'^2}\right] = 0 \tag{2.5}$$

$$\frac{\partial U'}{\partial X'} + \frac{\partial V'}{\partial Y'} = 0 \tag{2.6}$$

where U', V' are the velocity components along the X' and Y' directions respectively, P' is the pressure and μ is the viscosity of the fluid.

Taking into account the longitudinal motility of the walls, the boundary conditions are :

$$\left.\begin{aligned} U'(X',t') &= G_1'(X'-Ct'+V_p't') \\[2ex] V'(X',t') &= \frac{\partial H_1'}{\partial t'} \end{aligned}\right\} \quad \text{at } Y' = H_1'(X',t') \tag{2.7}$$

$$\left.\begin{aligned} U'(X',t') &= G_2'(X'-Ct'+V_p't') \\[2ex] V'(X',t') &= \frac{\partial H_2'}{\partial t'} \end{aligned}\right\} \quad \text{at } Y' = H_2'(X',t') \tag{2.8}$$

$$\left.\begin{aligned} U'(X',t') &= -V_p' \\[2ex] V'(X',t') &= \frac{\partial H_o'}{\partial t'} \end{aligned}\right\} \quad \text{at } Y' = H_o'(X',t') \tag{2.9}$$

where G_1' and G_2' represent the total longitudinal velocity of the walls.

Further the sheet is self-propelling and the forces exerted, by the fluid, on it must balance for its motion with a constant velocity, i.e.

$$\int(\tau^+ + \tau^-)dS = 0 \tag{2.10}$$

where τ^+ and τ^- are the resultant of forces acting on the upper and lower surfaces of the sheet respectively.

In a frame (x', y', t') moving with velocity $C-V_p'$ in the positive axial direction, the sheet and the walls of the channel appear stationary and the flow in this moving frame steady.

Using the following transformations

$$x' = X' - Ct' + V_p' t' \; ; \quad y' = Y' \tag{2.11}$$

and introducing the following dimensionless quantities

$$x = x'/\lambda, \; y = y'/a, \; t = Ct'/\lambda, \; p = p'/\frac{\mu C\lambda}{a^2}$$

$$(h_o, h_1, h_2, \varepsilon_o, \varepsilon_1, \varepsilon_2) = (h_o', h_1', h_2', b_o, b_1, b_2)/a,$$

$$(u, g_1, g_2, V_p) = (u', g_1', g_2', V_p')/C, \; v = v'/\frac{Ca}{\lambda}$$

(where symbols denoted by small letters in the moving frame represent the same quantities as denoted by capital letters in the fixed frame) we can write dimensionless form of eqns. (2.4)-(2.6) in the moving frame as follows

$$-\frac{\partial p}{\partial x} + \beta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \tag{2.12}$$

$$-\frac{\partial p}{\partial y} + \beta^2 \left(\beta^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) = 0 \tag{2.13}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.14}$$

where, $\qquad \beta = a/\lambda.$ $\hfill$ (2.15)

The boundary conditions for u are written in dimensionless form as

$$u = V_p + g_1(x) - 1 \quad ; \quad y = h_1(x)$$
$$u = V_p + g_2(x) - 1 \quad ; \quad y = h_2(x) \tag{2.16}$$
$$u = -1 \quad\quad\quad\quad\quad ; \quad y = h_o(x)$$

where

$$h_o(x) = \varepsilon_o \sin 2\pi x$$
$$h_1(x) = 1 + \varepsilon_1 \sin 2\pi x \tag{2.17}$$
$$h_2(x) = -1 + \varepsilon_2 \sin 2\pi x$$

For long wavelength approximation ($\beta \ll 1$) the eqns. (2.12) and (2.13) (after neglecting the terms involving β^2 and higher powers) are reduced to

$$- \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} = 0 \tag{2.18}$$

$$- \frac{\partial p}{\partial y} = 0 \tag{2.19}$$

Further using the stress-strain relationship to obtain the stresses in terms of u and v and applying long wavelength approximation, the force equilibrium condition (2.10) can be written as

$$\int_o^1 [p] \, dx = 0 \tag{2.20}$$

$$\int_o^1 \left([\frac{\partial u}{\partial y}]_{y=h_o(x)} + \frac{dh_o}{dx} [p] \right) dx = 0 \tag{2.21}$$

where $[f]$ indicates the difference in the quantity f above and below the sheet .

The equation (2.18) and (2.21) are solved in the following section, using conditions (2.16), to obtain the propelling velocity of the sheet and the total fluid flux in the channel.

2.3 MATHEMATICAL ANALYSIS

The differential equation (2.18) is solved with boundary conditions (2.17) to obtain the velocities u^+ and u^- in the regions $(h_o \leq y \leq h_1)$ and $(h_2 \leq y \leq h_o)$ respectively, which are given as follows

$$u^+ = \frac{1}{2}(\frac{\partial p^+}{\partial x})(y^2 - h_1 y - h_o y + h_1 h_o) + u_1 \frac{(y - h_o)}{h_1 - h_o} + u_o \frac{(h_1 - y)}{(h_1 - h_o)} \qquad (2.22)$$

$$u^- = \frac{1}{2}(\frac{\partial p^-}{\partial x})(y^2 - h_2 y - h_o y + h_o h_2) + u_2 \frac{(y - h_o)}{h_2 - h_o} + u_o \frac{(h_2 - y)}{(h_2 - h_o)} \qquad (2.23)$$

The dimensionless flow flux $q(= q'/ac)$ in the moving frame can be obtained as

$$q^+ = \int_{h_o}^{h_1} u^+ \, dy$$

$$q^- = \int_{h_2}^{h_o} u^- \, dy$$

which on using eqns. (2.22) and (2.23) give

$$q^+ = -\frac{1}{12}(\frac{\partial p^+}{\partial x})(h_1 - h_o)^3 + \frac{1}{2}(h_1 - h_o)(u_1 + u_o) \qquad (2.24)$$

$$q^- = -\frac{1}{12}(\frac{\partial p^-}{\partial x})(h_o - h_2)^3 + \frac{1}{2}(h_o - h_2)(u_2 + u_o) \qquad (2.25)$$

To obtain the pressure gradients $\frac{\partial p^+}{\partial x}$, the eqns. (2.24)

and (2.25) can be rewritten as

$$\frac{\partial p^+}{\partial x} = -12 \frac{q^+}{(h_1-h_o)^3} + 6 \frac{u_1+u_o}{(h_1-h_o)^2} \qquad (2.26)$$

$$\frac{\partial p^-}{\partial x} = -12 \frac{q^-}{(h_o-h_2)^3} + 6 \frac{u_o+u_2}{(h_o-h_2)^2} \qquad (2.27)$$

Integrating equation of continuity (2.14) it can be seen that the two fluxes $q^{\pm}$ are constants. Further from eqn. (2.19) it is evident that the pressure rise over a wavelength Δp is same for the two regions.

Integrating eqns. (2.26) and (2.27) over a wavelength, we get the following two relations among the three unknown quantities V_p, q^+ and q^-

$$-12q^+I_{11} + 6V_pI_{12} + 6I_{13} - 12I_{12} = \Delta p \qquad (2.28)$$

$$-12q^-I_{21} + 6V_pI_{22} + 6I_{23} - 12I_{22} = \Delta p \qquad (2.29)$$

Using the expressions (2.22), (2.23) and (2.26), (2.27) for $u^{\pm}$ and $\frac{\partial p^{\pm}}{\partial x}$, the force equilibrium condition (2.21) provide us the third relation, necessary to determine V_p, q^+ and q^-, as

$$12q^+I_{31} - 12q^-I_{32} + 2V_pI_{33} + 2(I_{34}+I_{35}) - 12I_{36} = 2\Delta p \qquad (2.30)$$

where

$$I_{11} = \int_o^1 \frac{dx}{(h_1-h_o)^3} \quad ; \quad I_{21} = \int_o^1 \frac{dx}{(h_o-h_2)^3}$$

$$I_{12} = \int_o^1 \frac{dx}{(h_1-h_o)^2} \quad ; \quad I_{22} = \int_o^1 \frac{dx}{(h_o-h_2)^2} \qquad (2.31)$$

$$I_{13} = \int_o^1 \frac{g_1(x)}{(h_1-h_o)^2} dx \quad ; \quad I_{23} = \int_o^1 \frac{g_2(x)}{(h_o-h_2)^2} dx$$

and

$$I_{31} = \int_o^1 \frac{h_1+h_o-1}{(h_1-h_o)^3} \, dx$$

$$I_{32} = \int_o^1 \frac{h_2+h_o+1}{(h_o-h_2)^3} \, dx$$

$$I_{33} = \int_o^1 \left[\frac{3-2h_1-4h_o}{(h_1-h_o)^2} + \frac{3+2h_2+4h_o}{(h_o-h_2)^2} \right] dx$$

$$I_{34} = \int_o^1 \left[\frac{3-2h_1-4h_o}{(h_1-h_o)^2} \right] g_1(x) \, dx$$

$$I_{35} = \int_o^1 \left[\frac{3+2h_2+4h_o}{(h_o-h_2)^2} \right] g_2(x) \, dx \qquad (2.32)$$

$$I_{36} = \int_o^1 \left[\frac{1-h_1-h_o}{(h_1-h_o)^2} + \frac{1+h_2+h_o}{(h_o-h_2)^2} \right] dx$$

Solving equations (2.28)-(2.30) we get

$$V_p = \frac{\left[(2+ \frac{I_{31}}{I_{11}} - \frac{I_{32}}{I_{21}}) \Delta p -2I_{34}-2I_{35}+12I_{36}- \frac{I_{31}}{I_{11}}(6I_{13}-12I_{12}) + \frac{I_{32}}{I_{21}}(6I_{23}-12I_{22}) \right]}{\left[2I_{33} + 6\frac{I_{31}}{I_{11}}I_{12} - 6\frac{I_{32}}{I_{21}} I_{22} \right]} \qquad (2.33)$$

and the total flux $q = q^+ + q^-$ as

$$q = -(\frac{1}{I_{11}} + \frac{1}{I_{21}}) \frac{\Delta p}{12} + (\frac{I_{12}}{I_{11}} + \frac{I_{22}}{I_{21}}) \frac{V_p}{2} + (\frac{I_{13}-2I_{12}}{2I_{11}} + \frac{I_{23}-2I_{22}}{2I_{21}})$$

$$(2.34)$$

The flux in the stationary frame Q is related with the

flux q in the moving frame by the following relations

$$Q = q + (1-V_p)(H_2-H_1) \tag{2.35}$$

Averaging equation (2.35) over a time period we, therefore, get the time averaged flow $\bar{Q}$ in terms of q and V_p as

$$\bar{Q} = q + 2(1-V_p) \tag{2.36}$$

Eqns. (2.31)-(2.36) provide expressions for V_p, q and $\bar{Q}$ for the given forms of $h_o(x), h_1(x), h_2(x)$ (eqn.2.17) and general forms of functions $g_1(x), g_2(x)$ representing the longitudinal motility of the channel. In the following analysis we consider the following forms of longitudinal motion of the wall:

$$g_1(x) = C_{11} + C_{12} \sin 2\pi x$$
$$g_2(x) = C_{21} + C_{22} \sin 2\pi x \tag{2.37}$$

For these given forms of h_o, h_1, h_2 and g_1, g_2 and writing

$$\alpha_1 = \varepsilon_1-\varepsilon_o, \quad \alpha_2 = \varepsilon_o-\varepsilon_2 \tag{2.38}$$

The expressions (2.31) and (2.32) can be rewritten as

$$I_{i1} = \int_o^1 \frac{dx}{(1+\alpha_i \sin 2\pi x)^3}$$

$$I_{i2} = \int_o^1 \frac{dx}{(1+\alpha_i \sin 2\pi x)^2} \quad (i=1,2) \tag{2.39}$$

$$I_{i3} = C_{i1}I_{i2} + C_{i2}I_{i4}$$

$$I_{i4} = \int_o^1 \frac{\sin 2\pi x}{(1+\alpha_i \sin 2\pi x)^2} dx$$

and

$$I_{31} = (\varepsilon_o + \varepsilon_1) \int_o^1 \frac{\sin 2\pi x}{(1+\alpha_1 \sin 2\pi x)^3} dx$$

$$I_{32} = (\varepsilon_o + \varepsilon_2) \int_o^1 \frac{\sin 2\pi x}{(1+\alpha_2 \sin 2\pi x)^3} dx$$

$$I_{33} = I_{12} - 2(\varepsilon_1 + 2\varepsilon_o) I_{14} + I_{22} + 2(\varepsilon_2 + 2\varepsilon_o) I_{24}$$

$$I_{34} = [I_{12} - 2(\varepsilon_1 + 2\varepsilon_o) I_{14}] C_{11} + [I_{14} - 2(\varepsilon_1 + 2\varepsilon_o) I_{37}] C_{12}$$

$$I_{35} = [I_{22} + 2(\varepsilon_2 + 2\varepsilon_o) I_{24}] C_{21} + [I_{24} + 2(\varepsilon_2 + 2\varepsilon_o) I_{38}] C_{22}$$

$$I_{36} = -(\varepsilon_1 + \varepsilon_o) I_{14} + (\varepsilon_2 + \varepsilon_o) I_{24}$$

$$I_{37} = \int_o^1 \frac{\sin^2 2\pi x}{(1+\alpha_1 \sin 2\pi x)^2} dx$$

$$I_{38} = \int_o^1 \frac{\sin^2 2\pi x}{(1+\alpha_2 \sin 2\pi x)^2} dx \qquad (2.40)$$

These integrals can be evaluated by contour integration and are listed in appendix 2(A).

In the following, some particular cases are obtained by using the following relations in the integrals, for integer m,n

$$\int_o^1 \frac{dx}{(1+\alpha \sin 2\pi x)^n} = \int_o^1 \frac{dx}{(1-\alpha \sin 2\pi x)^n} \qquad (2.41)$$

and $$\int_o^1 \frac{\sin^m 2\pi x \, dx}{(1+\alpha \sin 2\pi x)^n} = (-1)^m \int_o^1 \frac{\sin^m 2\pi x \, dx}{(1-\alpha \sin 2\pi x)^n}$$

When there is no peristaltic motion of the walls, we can write

$$\alpha_1 = -\varepsilon_o , \quad \alpha_2 = \varepsilon_o$$

and using relations (2.41) we obtain the following relations in

the integrals given by (2.39) and (2.40)

$$I_{11} = I_{21}$$

$$I_{12} = I_{22}$$

$$I_{13} = C_{11}I_{12} + C_{12}I_{14}$$

$$I_{23} = C_{21}I_{12} - C_{22}I_{14}$$

$$I_{14} = -I_{24}$$

$$I_{31} = -I_{32}$$

$$I_{33} = 2I_{12} - 8\varepsilon_o I_{14}$$

$$I_{34} = C_{11}(I_{12} - 4\varepsilon_o I_{14}) + C_{12}(I_{14} - 4\varepsilon_o I_{37})$$

$$I_{35} = C_{21}(I_{12} - 4\varepsilon_o I_{14}) - C_{22}(I_{14} - 4\varepsilon_o I_{37})$$

$$I_{36} = -2\varepsilon_o I_{14}$$

$$I_{37} = I_{38} \tag{2.42}$$

We therefore get the expressions for V_p, q and $\bar{Q}$ from equations (2.33), (2.34), (2.36) as,

$$V_p = \frac{1}{M_o}(M_1 \Delta p + M_2) \tag{2.43}$$

$$q = \frac{1}{6}\Delta p(-\frac{1}{I_{11}} + \frac{6M_1}{M_o}\frac{I_{12}}{I_{11}}) + \frac{M_2}{M_o}(\frac{I_{12}}{I_{11}}) + \frac{1}{2I_{11}}[I_{12}(C_{11}+C_{21}) + I_{14}(C_{12}-C_{22})$$

$$-4I_{12}] + \frac{1}{2I_{11}}[I_{12}(C_{11}+C_{21}) + I_{14}(C_{12}-C_{22}) - 4I_{12}] \tag{2.44}$$

$$\bar{Q} = \frac{1}{6}\Delta p(-\frac{1}{I_{11}} + 6\frac{M_1}{M_o}\frac{I_{12}}{I_{11}} - 12\frac{M_1}{M_o}) + \frac{M_2}{M_o}(\frac{I_{12}}{I_{11}} - 2) + 2$$

$$+ \frac{1}{2I_{11}}[I_{12}(C_{11}+C_{21}) + I_{14}(C_{12}-C_{22}) - 4I_{12}] \tag{2.45}$$

where $M_0 = 4I_{12} - 16\varepsilon_0 I_{14} + 12 \dfrac{I_{31}}{I_{11}} I_{12}$

$M_1 = 2 + 2 \dfrac{I_{31}}{I_{11}}$

$M_2 = -2(I_{12}-4\varepsilon_0 I_{14})(C_{11}+C_{21}) - 2(I_{14}-4\varepsilon_0 I_{37})(C_{12}-C_{22})$

$\qquad -24\varepsilon_0 I_{14} - 6 \dfrac{I_{31}}{I_{11}} [I_{12}(C_{11}+C_{21}) + I_{14}(C_{12}-C_{22})]$

$$\tag{2.46}$$

Further if longitudinal motion is also absent i.e.

$$g_1(x) = 0 , \quad g_2(x) = 0$$

we get a case when the sheet is swimming through a channel.
Using the values of the integrals from appendix 2(A), we get
eqns. (2.43)-(2.45) reduced to

$$V_p = \frac{1}{2}(1-\varepsilon_0^2)^{1/2} \Delta p + \frac{3\varepsilon_0^2}{1+2\varepsilon_0^2}$$

$$q = \frac{1}{3}(1-\varepsilon_0^2)^{3/2} \Delta p - \frac{2(1-\varepsilon_0^2)}{1+2\varepsilon_0^2} \tag{2.47}$$

$$\bar{Q} = -\frac{1}{3}(2+\varepsilon_0^2)(1-\varepsilon_0^2)^{1/2} \Delta p$$

These expressions are same as obtained by Shack and
Lardner (1974) for $q = 2Q$ and $\bar{Q} = 2Q_F$. Thus the proposed model
includes the case studied by Shack and Lardner (1974) as a
particular case when there is no peristaltic and longitudinal
motion of the walls of the channel.

2.4 RESULTS AND DISCUSSIONS

The values of V_p and $\bar{Q}$ have been computed using eqns. (2.33)-
(2.36) for the following set of parameters

$$C_{11} = 0 \quad , \quad C_{21} = 0, \quad C_1(=C_{12}) = (0,0.2,-0.2), C_2(=C_{22})=(0,0.2,-.2)$$
$$\varepsilon_1 = (0,0.35), \quad \varepsilon_2 = (0,0.25,-0.25), \Delta p = (0,.05,-.05)$$

and are plotted in figs 2.2 - 2.13 as functions of ε_o.

Effects of various parameters on V_p is graphically shown in fig. 2.2 - 2.7. In fig. 2.2 the results corresponding to the particular case, when there is no peristaltic motion on the walls ($\varepsilon_1 = 0$, $\varepsilon_2 = 0$), are shown. As noted in the expressions (2.43)-(2.46) the velocity amplitudes $C_1(=C_{12})$ and $C_2(=C_{22})$ appear always as their difference $C_2-C_1=d$. The effect of this parameter d is therefore, shown for $\Delta p = 0,.05$. It is observed that V_p increases as magnitude of ε_o increases. For fixed $\varepsilon_o > 0$, V_p decreases as d increases and for $\varepsilon_o < 0$, a reverse trend is noticed. It is also noted that, in this particular case, there is no effect of the longitudinal velocities C_1, C_2 if $\varepsilon_o = 0$ which is also evident from the expressions (2.43) and (2.46). The effect of Δp, for fixed values of ε_o and d is to increase V_p.

Effect of C_1 and C_2 for non-zero values of $\varepsilon_1, \varepsilon_2$ is shown in fig. 2.3 for $\Delta p = 0$. The effect of C_1 and C_2 is similar as observed in fig. 2.2. Comparing fig. 2.2 and 2.3, it can also be noticed that because of the peristalsis on the walls, the propagation velocity V_p is reduced and, in fact, is in the opposite direction. The maximum negative value of V_p is obtained for the combination $C_1 < 0$, $C_2 > 0$.
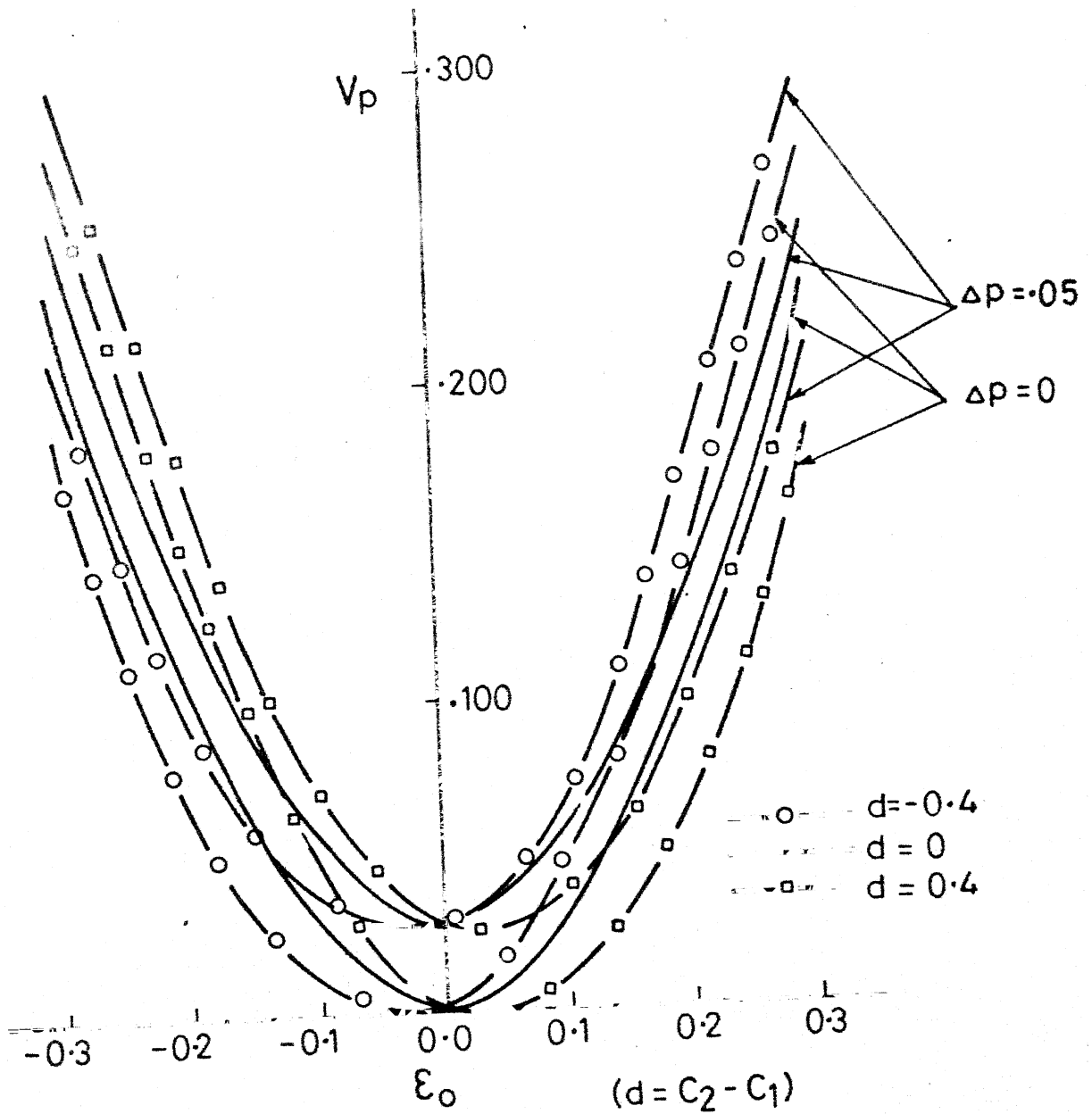
Fig. 2·2 Effect of C_1 and C_2 on V_p for zero ε_1 & ε_2 (no peristalsis)
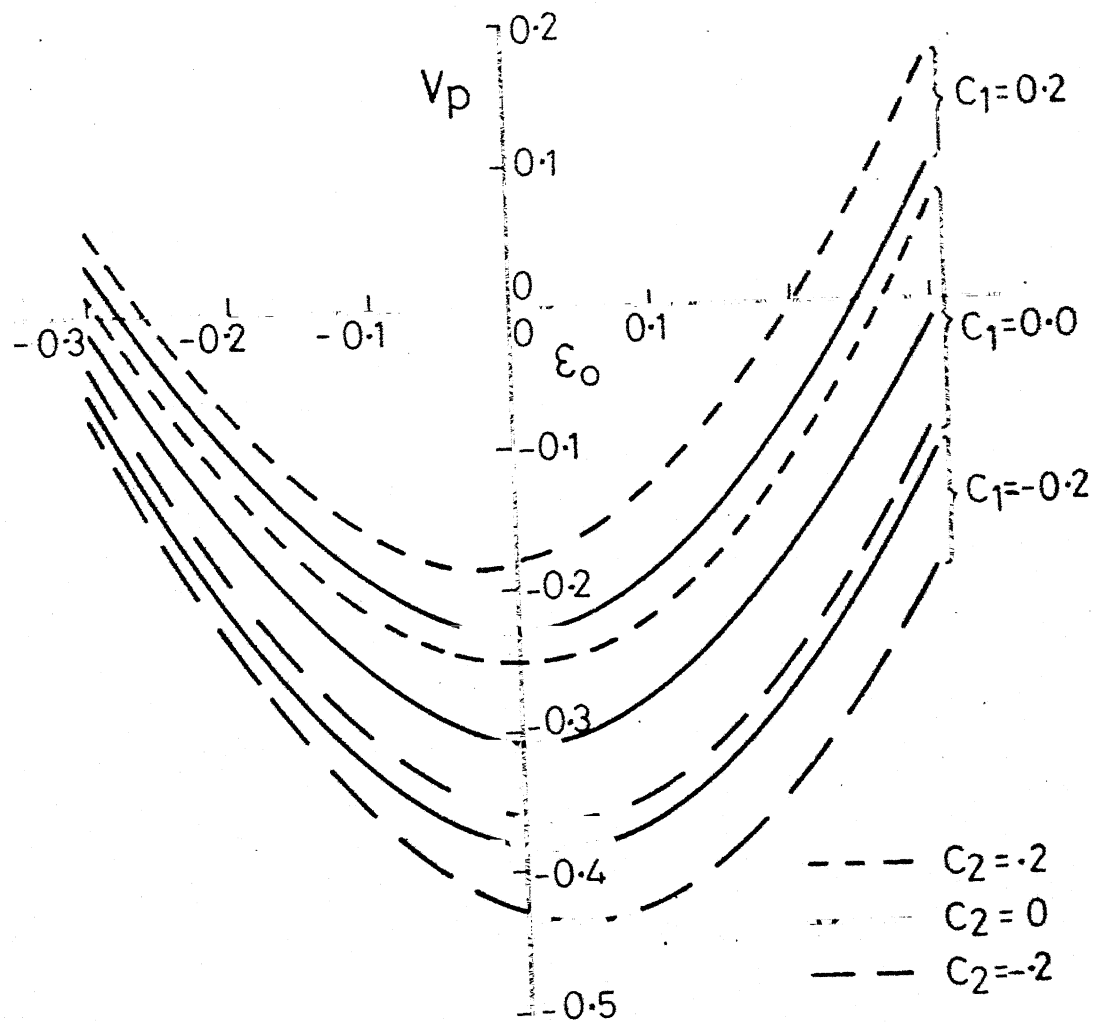
Fig.2·3 Effect of C_1 and C_2 on V_p for fixed ε_1 & ε_2 ($\varepsilon_1 = 0.35$, $\varepsilon_2 = 0.25$ and $\Delta p = 0$)
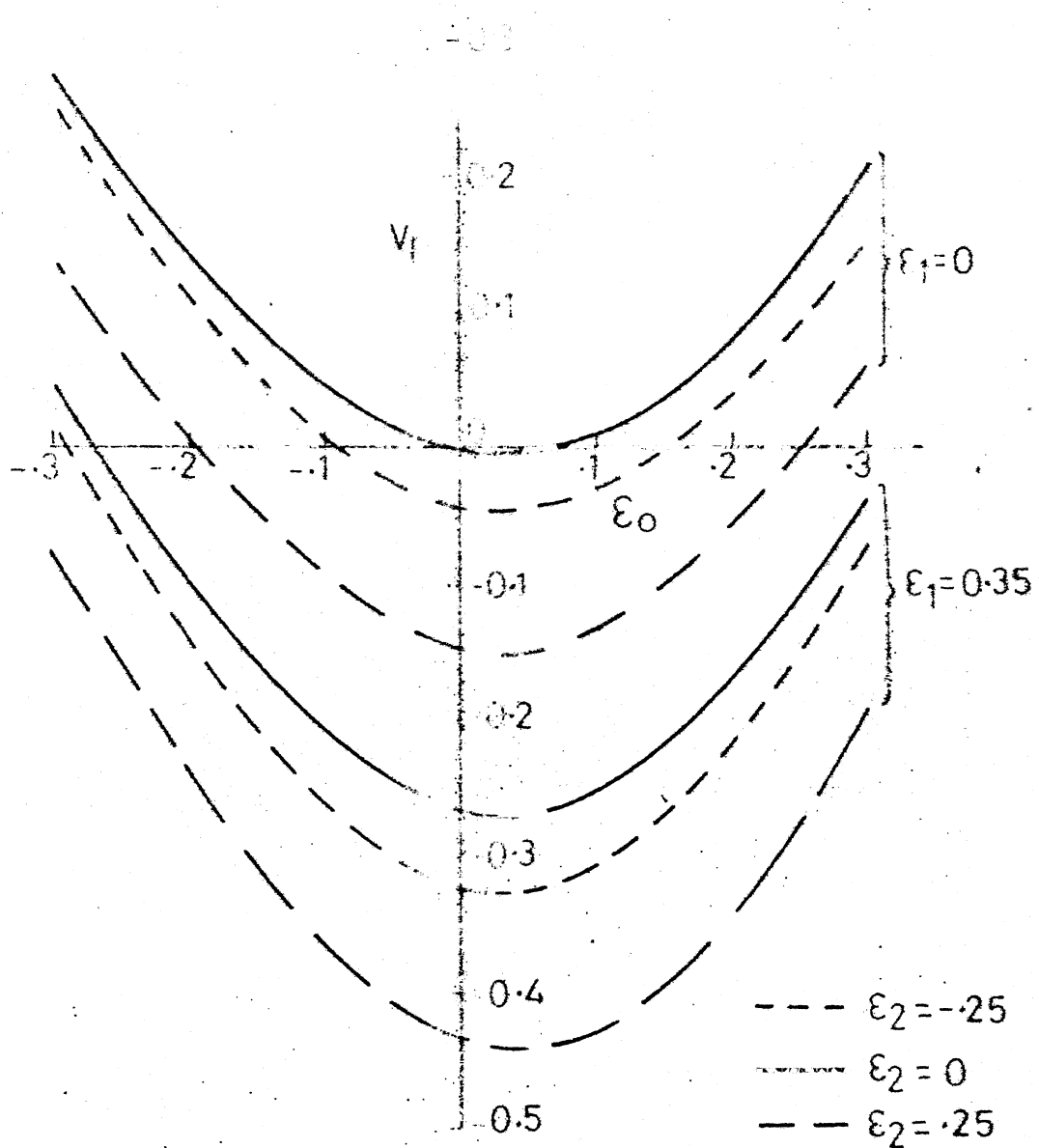
Fig. 2.4 Effect of ε_1 and ε_1 on V_p ($C_1 = -2$, $C_2 = 0.2$, $\Delta p = 0$)
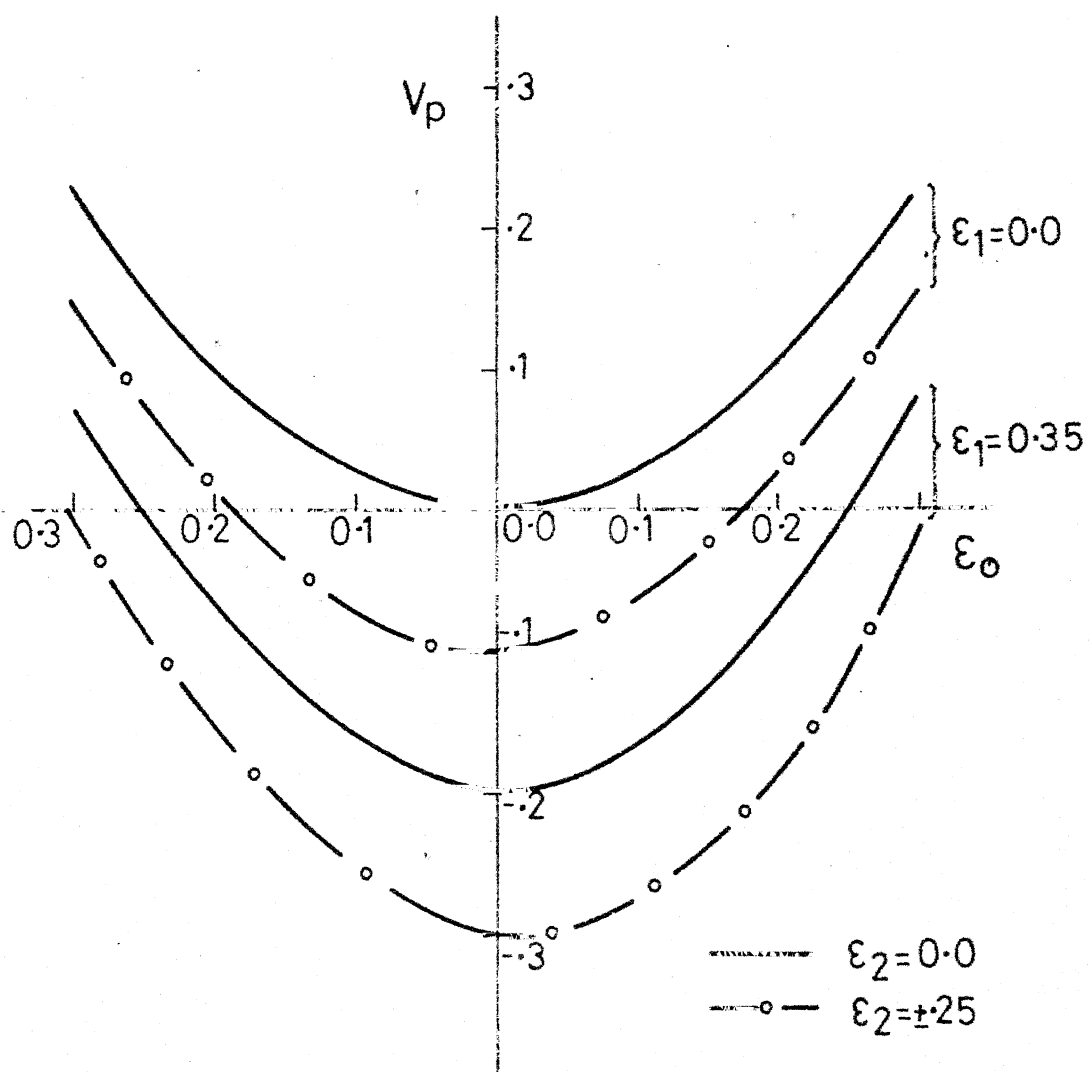
Fig. 2·5 Effect of ε_1 & ε_2 on V_p
$(C_1 = 0, C_2 = 0, \Delta p = 0)$
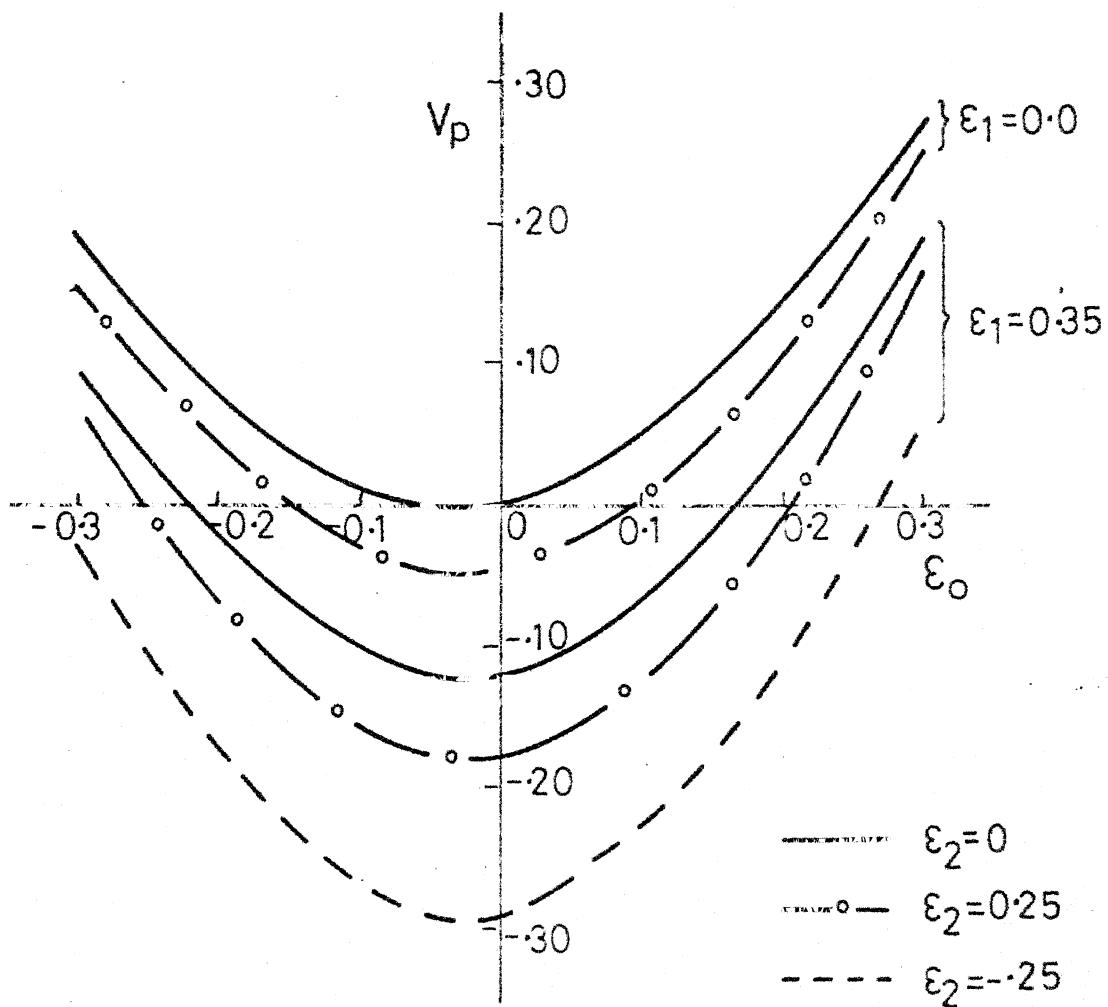
Fig. 2·6 Effect of ε_1 & ε_2 on V
$(C_1 = \cdot 2, C_2 = \cdot \cdot 2, \triangle p = 0)$

Fig.2·7 Effect of Δp and $\varepsilon_1, \varepsilon_2$ on V_p
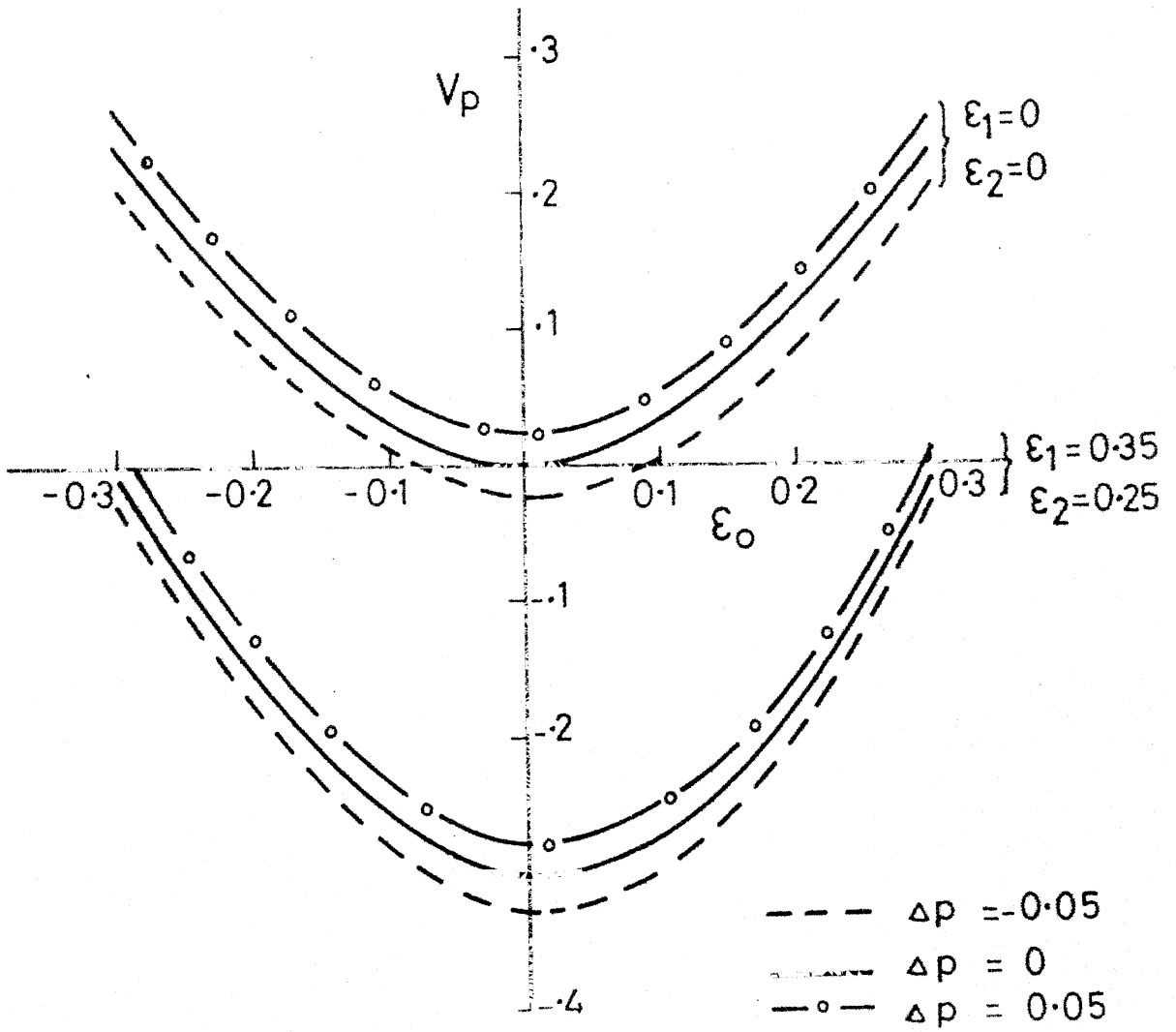$(C_1 = 0, C_2 = 0)$

Similar effects of $\varepsilon_1, \varepsilon_2, C_1$ and C_2 can be seen from fig. 2.4 - 2.6, where effects of ε_1 and ε_2 are shown for fixed $(C_1, C_2) = [(-.2, .2), (0, 0), (.2, -.2)]$ and $\Delta p = 0$. Comparing these three figures it can be observed that for $C_1 = C_2 = 0$, sign of ε_2 does not matter. As for the maximum negative value of V_p for fixed $\varepsilon_o, \varepsilon_1$ is concerned, it is obtained for the first case $(C_1 < 0, C_2 > 0)$ with $\varepsilon_2 > 0$ (fig. 2.4).

Increase in Δp, the pressure rise causes increase in V_p. A negative value of Δp, however, reduces the value of V_p to negative values (fig. 2.7).

Effect of various parameters on time-average flux $\bar{Q}$ is shown in fig. 2.8-2.13. It can be seen that in general the results are opposite to those obtained for V_p. The effect of peristaltic waves on the wall is to increase $\bar{Q}$ but increase in the magnitude of the amplitude ε_o of the causes a decrease in $\bar{Q}$, (fig. 2.8, 2.10, 2.11). It also increases as Δp becomes negative, (fig. 2.12, 2.13). For a particular case of no peristalsis $(\varepsilon_1 = \varepsilon_2 = 0)$ the effect of d is oppositie to that of V_p i.e. $\bar{Q}$ is minimum for $d > 0$, (fig.2.8). However in general case maximum value of $\bar{Q}$ is obtained when C_1 and C_2 are both negative (fig. 2.9).

2.5 MOTION OF A THIN SHEET IN A CHANNEL UNDER PERISTALTIC MOTION OF THE WALLS

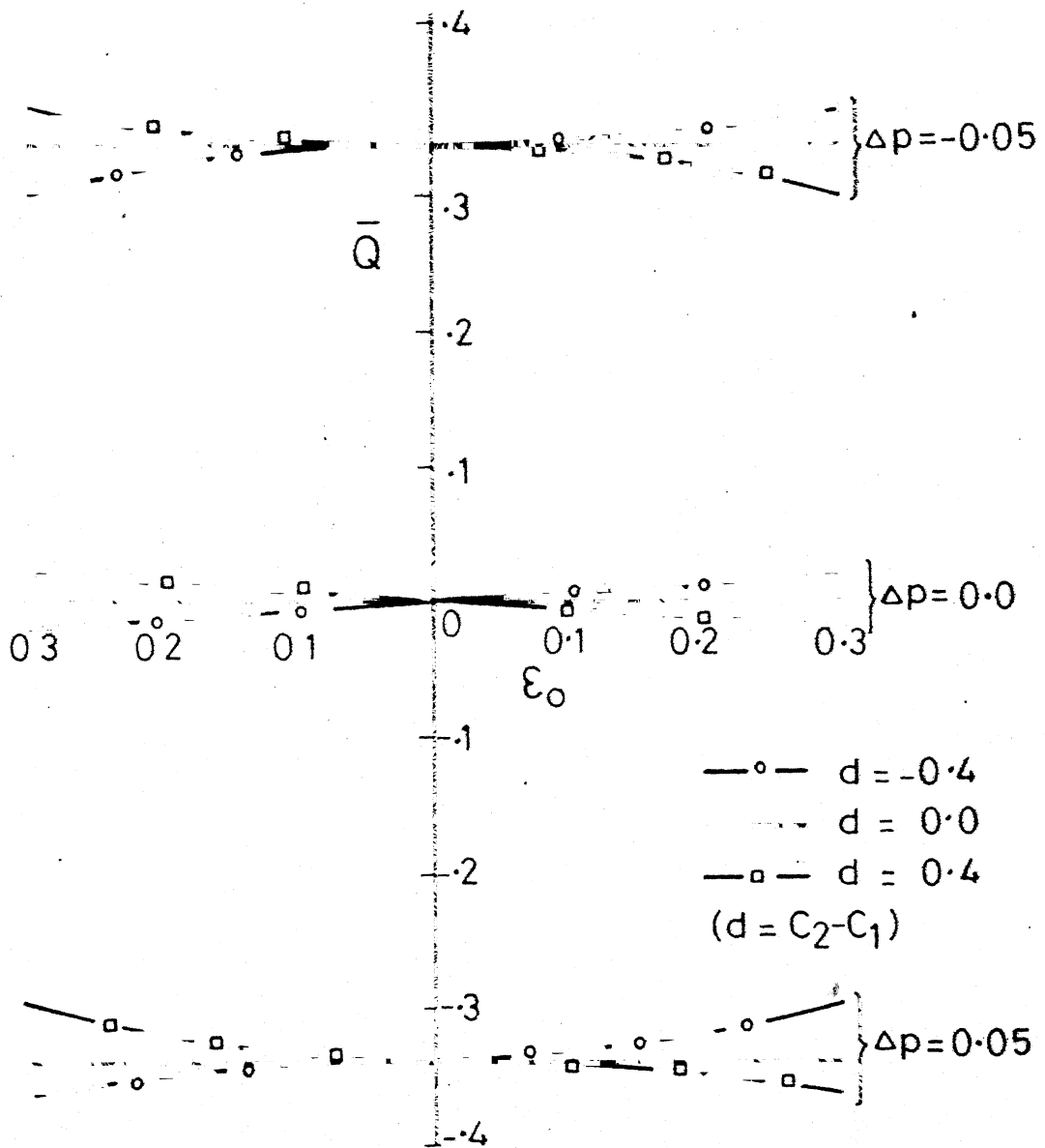This case can be obtained from the present model when
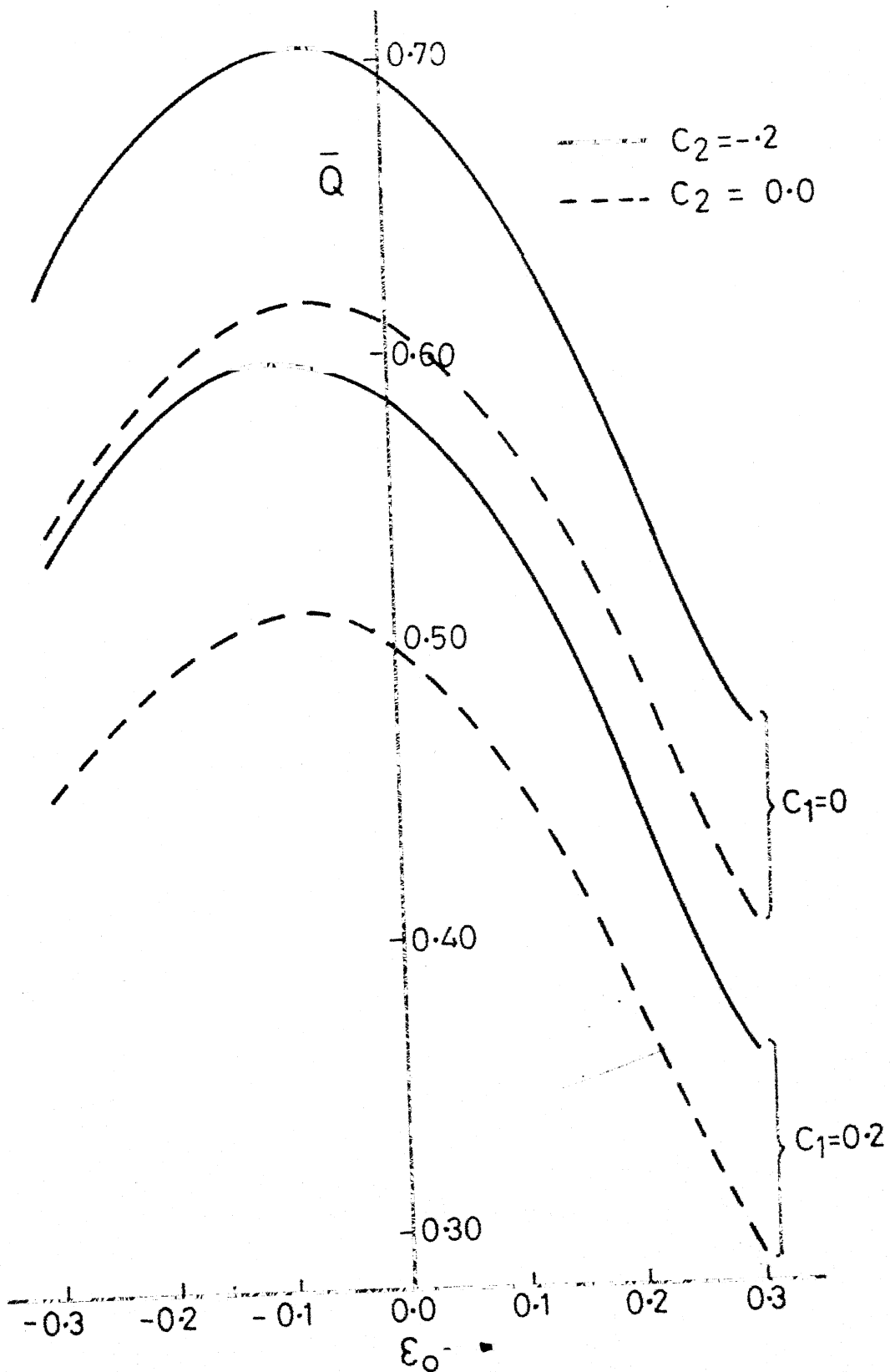
Fig.2·8 Effect of C_1 and C_2 on $\bar{Q}$ for zero
ε_1 & ε_2 (no peristalsis)

Fig. 2.9 Effect of C_1 and C_2 on $\bar{Q}$
$\varepsilon_1 = \cdot 35$, $\varepsilon_2 = -\cdot 25$, $\triangle p = 0$
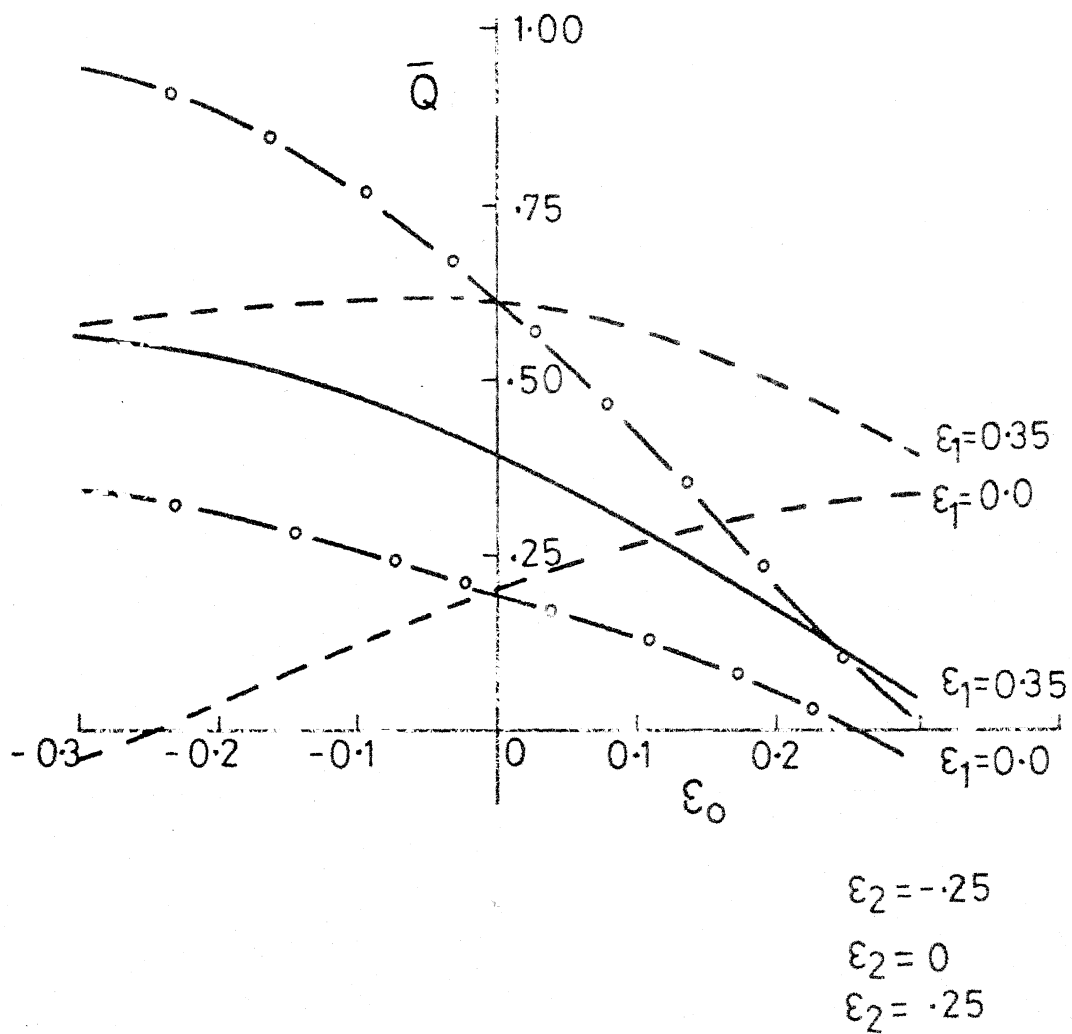
Fig. 2·10 Effect of ε_1 and ε_2 on $\bar{Q}$
$(C_1=0, C_2=0, \triangle p = 0)$
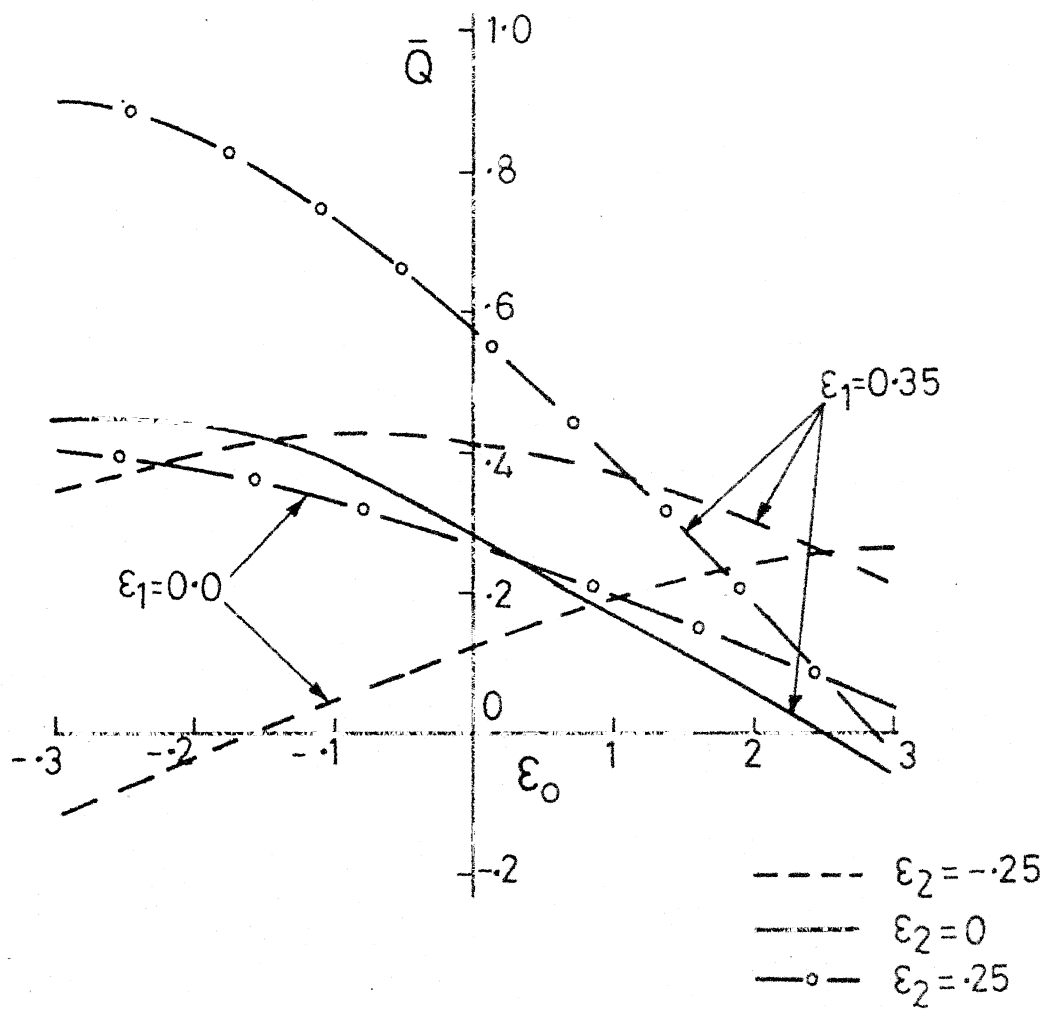
Fig. 2·11   Effect of ε_1 and ε_2 on $\bar{Q}$
$(C_1 = \cdot2, C_2 = -2 , \triangle p = 0)$
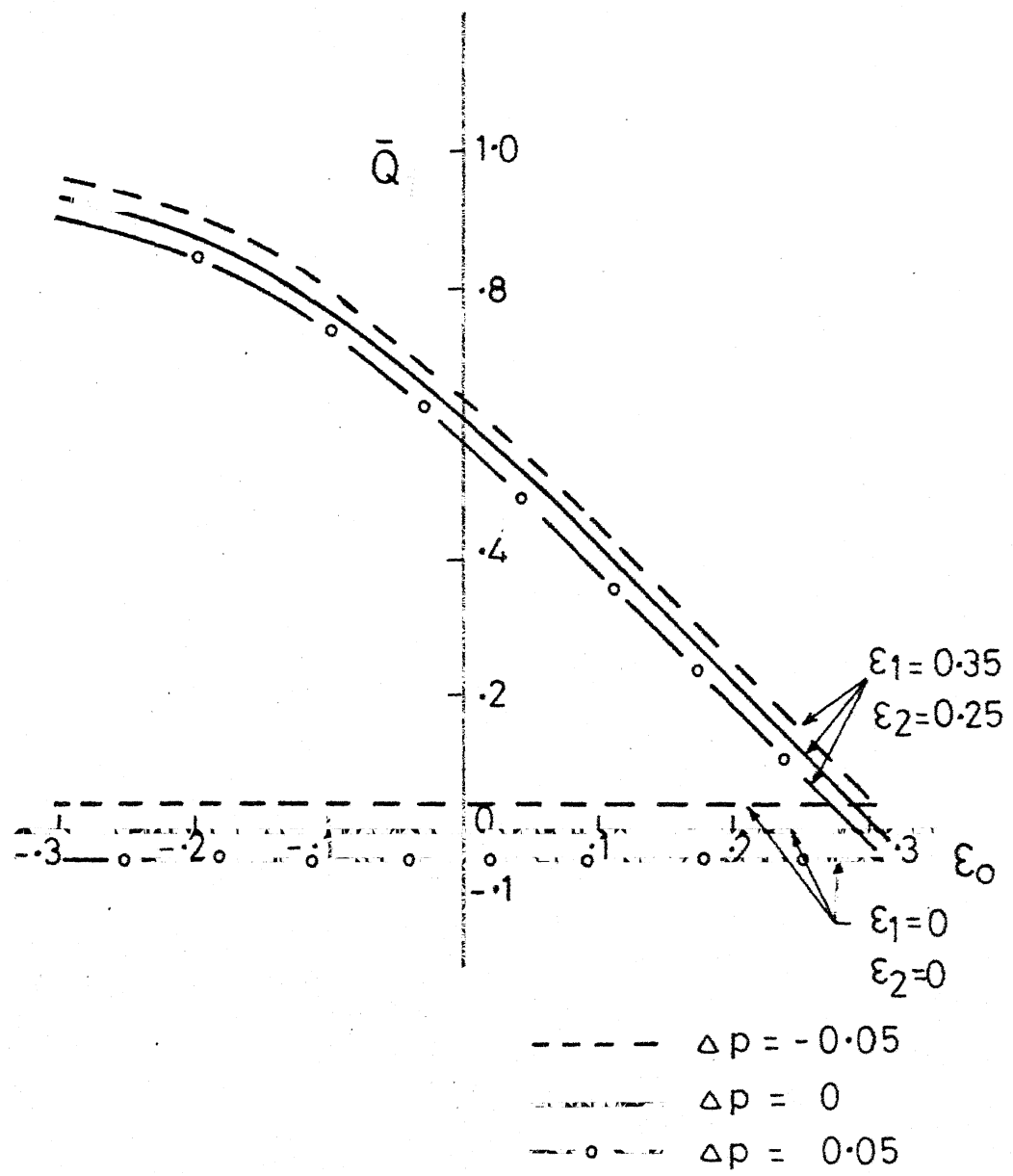
Fig. 2.12 Effect of Δp on $\bar{Q}$ ($C_1=0, C_2=0$)
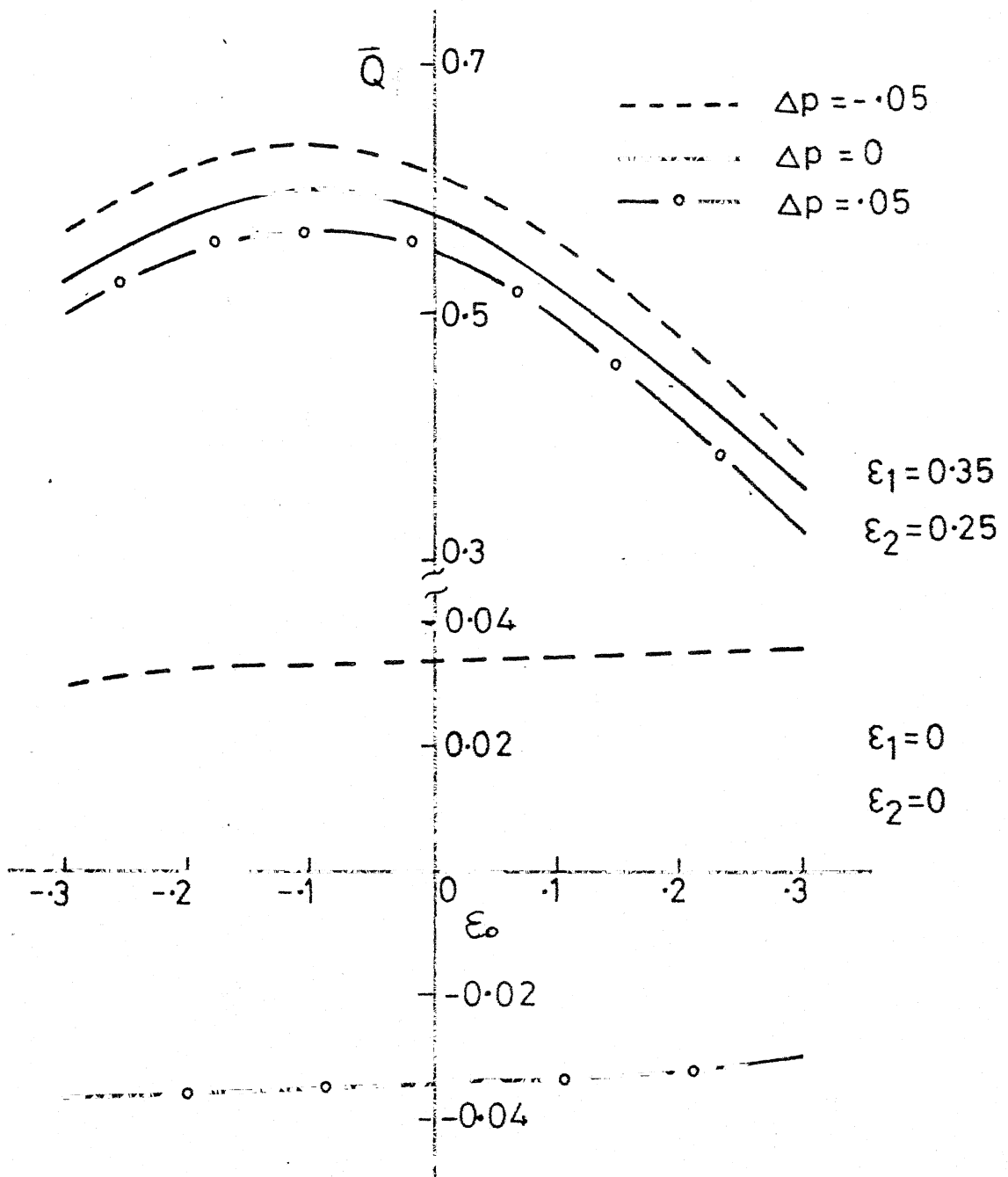
Fig. 2·13 Effect of Δp on $\bar{Q}$ ($C_1 = \cdot 2$, $C_2 = -\cdot 2$)

there are no lateral wave on the sheet and no longitudinal wave motion along the channel walls. It should be noted that this case is physically different from the usual peristaltic flow in a channel [Shapiro et al. (1969)] as there exists a sheet in the middle of the channel which divide it into two separate zones.

Mathematically, this case is obtained by putting $g_1(x) = 0$, $g_2(x) = 0$ and $\varepsilon_o = 0$, in the equations (2.33)-(2.36) and then the expressions for V_p, q and $\bar{Q}$ become (for the symmetric case when $\varepsilon_1 = -\varepsilon_2 = \varepsilon$)

$$V_p = \frac{1}{M_o}(M_1 \Delta p + M_2)$$

$$q = \frac{1}{6} \Delta p(-\frac{1}{I_{11}} + 6\frac{M_1 I_{12}}{M_o I_{11}}) + (-2\frac{I_{12}}{I_{11}} + \frac{M_2 I_{12}}{M_o I_{11}})$$

$$\bar{Q} = \frac{1}{6} \Delta p(-\frac{1}{I_{11}} + 6\frac{M_1 I_{12}}{M_o I_{11}} - 12\frac{M_1}{M_o}) + (-2\frac{I_{12}}{I_{11}} + 2 + \frac{M_2 I_{12}}{M_o I_{11}} - 2\frac{M_2}{M_o})$$

$$\tag{2.48}$$

where
$$M_o = 4I_{12} - 8\varepsilon I_{14}$$

$$M_1 = 2 + 2\frac{I_{31}}{I_{11}} \tag{2.49}$$

$$M_2 = -24 \varepsilon I_{14} + 24 \frac{I_{31}}{I_{11}} I_{12}$$

The effects of $\Delta p, \varepsilon_1$ and ε_2 on V_p and $\bar{Q}$ can be seen from figs. 2.2, 2.5, 2.8 and 2.12 at $\varepsilon_o = 0$. It can be noted from fig. 2.2 and 2.3 that the velocity V_p of the sheet increases as Δp increases. However for $\varepsilon_1 \neq 0$, $\varepsilon_2 \neq 0$ it is seen that V_p is negative, i.e. the sheet flow in the direction of

peristaltic waves and this effect is enhanced as ε_1 and ε_2 increase. The effects of Δp, ε_1 and $\dot\varepsilon_2$ on $\bar{Q}$ can be studied from figs. 2.8 and 2.12. It is observed that $\bar{Q}$ increases as the magnitude of ε_1 and ε_2 increase and as Δp decreases.

2.6 APPLICATION TO SPERMATOZOA TRANSPORT : AN APPROACH TO
    FERTILITY CONTROL

The spermatozoa are male reproductive cells. They are specialized cells containing only 23 out of full 46 chromosomes of a normal cell in human body. During coitus, the spermatozoa are deposited, at the time of ejaculation at the mouth of the cervix in femal genital tract. It has been found that they take on average 45 minute to reach the fertilization point in the fallopian tube and thus travel with an average speed of 1-3 m.m./ minute. It is necessary, from the point of fertility control, to reduce this speed of spermatozoa in the female genital tract by some biochemical and other means.

It is known that spermatozoa, when in large number and close to each other, travel in unison. It is therefore possible to approximate their motion by a sheet flowing through fluid [Taylor (1951)]. The proposed model and the results discussed in section 2.4, can therefore be applied to study the effects of peristaltic and longitudinal motion of the walls of the female genital tract on the spermatozoa transport.

Noting the effects of various parameters on V_p, as

discussed in section 2.4, it is evident that V_p can be made
negative by application of a negative Δp (i.e. pressure drop)
and this effect can further be enhanced by inducing the
peristaltic waves travelling along the walls in the direction
away from the point of fertilization. Further the motion of
the cilia can also enhance this effect by a suitable choice
of longitudinal wave. It is therefore suggested that if the
generation of pressure drop and the peristaltic motion is
possible by some mechanism, such as sudden withdrawl of male
organ from the vagina immediately after ejaculation, it may be
possible to reduce the motion of the spermatozoa and hence
controling fertility. Thus the ancient belief behind the
practise of withdrawl of the male organ immediately after
ejaculation, to control fertility, may have a scientific basis
as predicted by this model.

2.7 CONCLUSION :

A mathematical model to study the effect of peristaltic
and longitudinal motion of the walls, on the propulsion of a
sheet has been presented and the results have been applied
on the swimming of spermatozoa through female genital tract.
It has been shown that the speed of spermatozoa can be reduced
considerably, to avoid fertilization, by generating a pressure
drop and inducing peristaltic waves on the wall of the genital
tract through some biomechanical and other means such as
sudden withdrawl of male organ from the vagina, immediately
after ejaculation.

APPENDIX 2(A)

VALUES OF THE INTEGRALS FROM CONTOUR INTEGRATION METHODS

$$\int_0^1 \frac{dx}{(1+\alpha \sin 2\pi x)} = \frac{1}{(1-\alpha^2)^{1/2}}$$

$$\int_0^1 \frac{dx}{(1+\alpha \sin 2\pi x)^2} = \frac{1}{(1-\alpha^2)^{3/2}}$$

$$\int_0^1 \frac{dx}{(1+\alpha \sin 2\pi x)^3} = \frac{2+\alpha^2}{2(1-\alpha^2)^{5/2}}$$

$$\int_0^1 \frac{dx}{(1+\alpha \sin 2\pi x)^4} = \frac{2+3\alpha^2}{2(1-\alpha^2)^{7/2}}$$

$$\int_0^1 \frac{\sin 2\pi x \, dx}{(1+\alpha \sin 2\pi x)^2} = -\frac{\alpha}{(1-\alpha^2)^{3/2}}$$

$$\int_0^1 \frac{\sin 2\pi x \, dx}{(1+\alpha \sin 2\pi x)^3} = -\frac{3\alpha}{2(1-\alpha^2)^{5/2}}$$

$$\int_0^1 \frac{\sin 2\pi x \, dx}{(1+\alpha \sin 2\pi x)^4} = -\frac{5\alpha}{3(1-\alpha^2)^{7/2}} + \frac{\alpha}{3(1-\alpha^2)^{5/2}}$$

$$\int_0^1 \frac{\sin^2 2\pi x \, dx}{(1+\alpha \sin 2\pi x)^2} = -\frac{1}{\alpha^2} + \frac{2\alpha^2-1}{\alpha^2(1-\alpha^2)^{3/2}}$$

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CHAPTER III

MOTION OF MICRO-ORGANISMS UNDER PERISTALSIS : APPLICATION TO OVUM TRANSPORT

3.1 INTRODUCTION:

In the past few decades the fluid-dynamical aspects of peristaltic pumping have been extensively studied because of its importance in transporting fluids in many biological systems [Burns and Parkes (1967), Barton and Raynor (1968), Fung and Yih (1968), Shapiro et al. (1969), Chow (1970), Jaffrin and Shapiro (1971), Weinberg et al. (1971)]. A review of much of the literature regarding this has been presented by Jaffrin and Shapiro (1971). Most of these studies pertain to the motion of urine in ureter and chyme movement in intestines and very little attention has been paid to study the effect of peristalis on biosystems such as the transport of micro-organisms like spermatozoa and ovum. Further it is also known that cilia attached to the surface of transporting ducts, such as oviduct, play a significant role in transport of fluids in many situations [Barton and Raynor (1967), Lardner and Shack (1972)]. It would, therefore, be interesting to study these effects on the transport of micro-organism such as ovum.

The study of self-propelling micro-organisms was initiated by Taylor (1951) who assumed the organism to be an infinite sheet of zero thickness. Later Reynolds (1965)

extended this analysis to include the effects of inertia. Shack and Lardner (1974) also studied this problem under long wavelength approximation. Attempts were also made to explain the motion of spermatozoa in the female genital tract by considering the dynamical interaction of the wall [Smelser et al. (1973), Shukla et al. (1978)].

Recently Blake et al. (1983) have presented a theoretical model of ovum-transport in oviduct incorporating transport mechanisms due to ciliary and muscular activity by adding a force distribution term in the equation of motion. The motion due to ciliary and muscular activity can also be represented as a combination of transverse and longitudinal waves along the wall and effects of this interaction should be considered on ovum transport.

Keeping these in view, in this chapter models are proposed to study the effects of peristaltic and longitudinal wave motion along the wall of the duct (rectangular and circular) on the locomotion of micro-organisms of finite thickness which may also generate symmetrical peristaltic waves along their bodies. The governing equations of this system have been solved under long wavelength approximation and the effects of various parameters on the velocity of micro-organism and the flux, are studied. This analysis has also been applied to study the transport of ovum in oviduct. As one of the aims of the proposed study is to

suggest a biomechanical or biochemical means for fertility control, the interaction of peristaltic waves and longitudinal motion and pressure difference on the movement of ovum in the oviduct have been studied. It has been shown that if there are no transverse and longitudinal waves along the wall of the oviduct, the ovum may not be able to move forward for fertilization.

3.2 TRANSPORT THROUGH A CHANNEL (CASE 1)

3.2.1 MATHEMATICAL MODEL

Consider the propulsion of a flexible slab (micro-organism) of infinite width and finite thickness $2\delta'$ in the middle of a channel. The space between the boundaries of the slab and the channel is filled with a Newtonian fluid and the walls of the channel are undergoing longitudinal wave motion. It is assumed that the slab is being propelled at a constant speed V_p' in the negative X'-direction. Let travelling sinusoidal waves with speed $c-V_p'$ and wavelength λ but different amplitudes b' and b_1' travel along the surfaces of the channel and the micro-organism, respectively. In a stationary frame (X', Y', t') the shapes of the wall of the channel $(Y' = \pm H')$ and the micro-organism $(Y' = \pm H_1')$ at any instant t' are therefore given by (see Fig. 3.1)

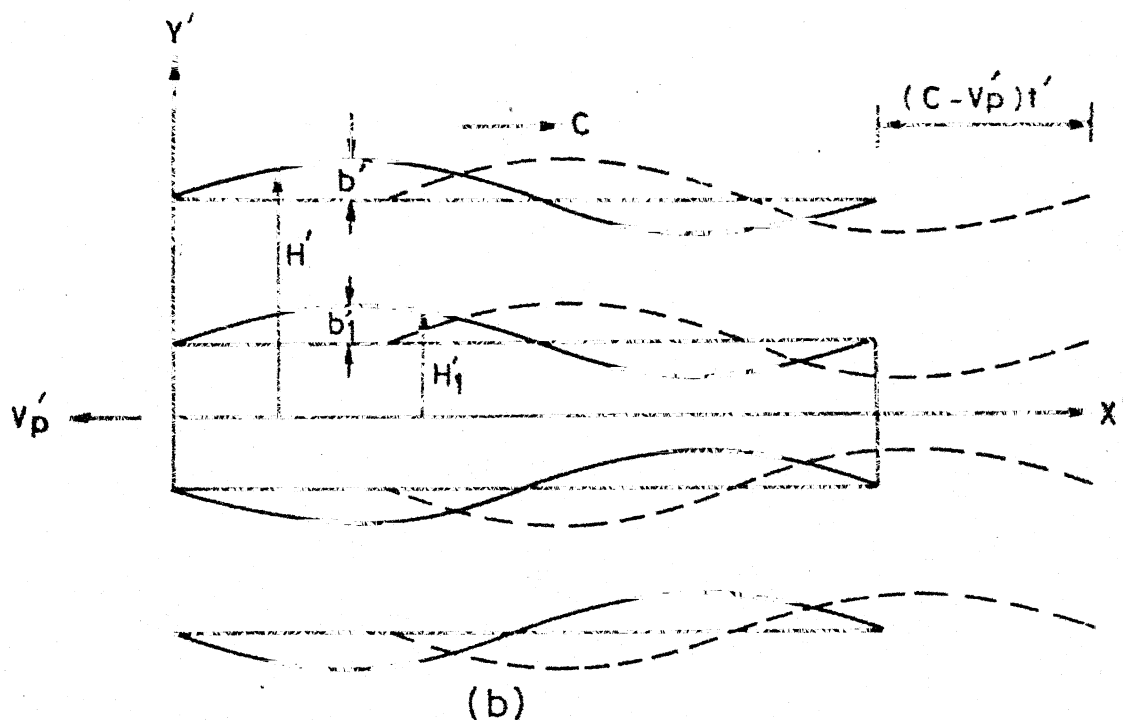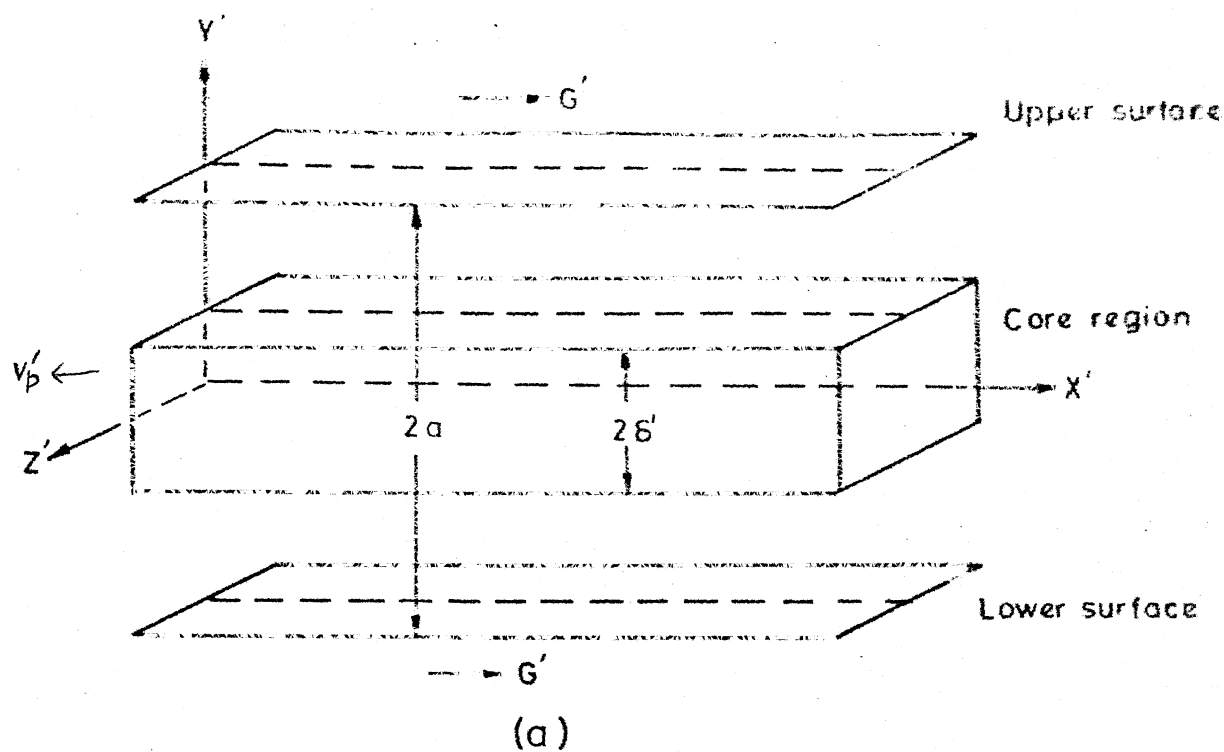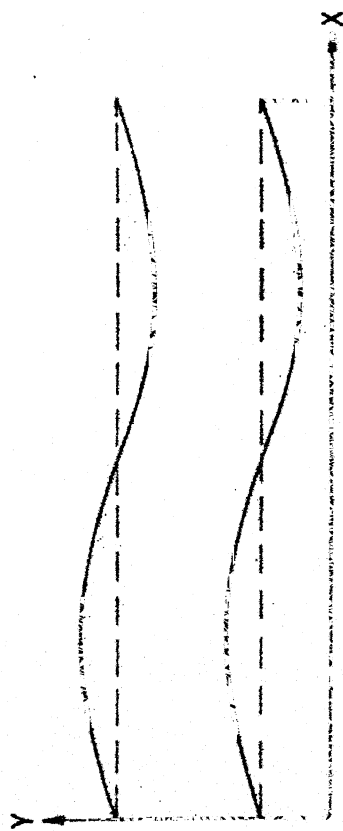$$H'(X', t') = a + b' \sin \frac{2\pi}{\lambda} (X' - ct' + V_p' t') \qquad (3.1)$$
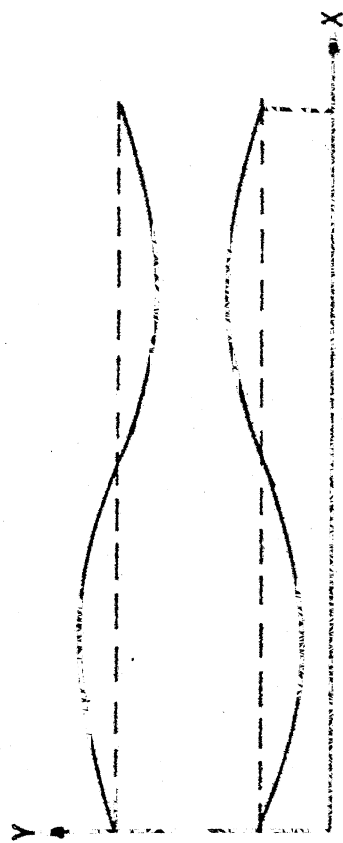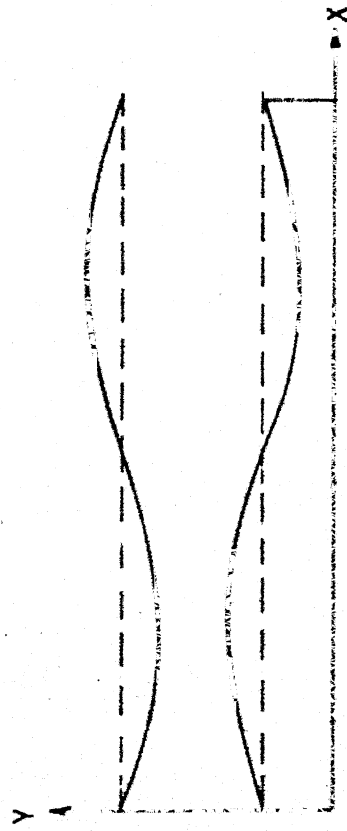
FIG. 3.1 SWIMMING OF A MICRO ORGANISM THROUGH A CHANNEL
(a) WITHOUT PERISTALSIS (b) WITH PERISTALSIS
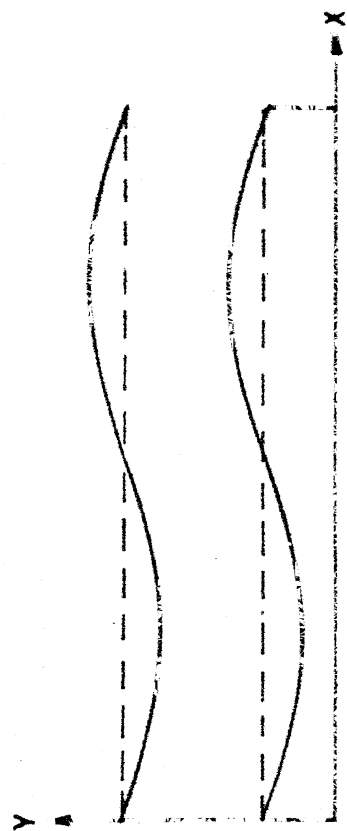
(i) $b > 0$ and $b_j' > 0$

(ii) $b > 0$ and $b_j' > 0$

(iii) $b > 0$ and $b_j' > 0$

(iv) $b > 0$ and $b_j' > 0$

FIG. 3.2   VARIOUS CASES (CHANNEL) (Only the region $y' > 0$ is shown)

$$H_1'(X', t') = \delta' + b_1' \sin \frac{2\pi}{\lambda}(X' - ct' + V_p t') \qquad (3.2)$$

Since the Reynolds number, as in Chapter II, is of order 10^{-3}, the equations governing the flow of the fluid in the gap between the boundaries of the microorganism and channel (after neglecting the inertia terms) are

$$-\frac{\partial P'}{\partial X'} + \mu \left(\frac{\partial^2 U'}{\partial X'^2} + \frac{\partial^2 U'}{\partial Y'^2}\right) = 0 \qquad (3.3)$$

$$-\frac{\partial P'}{\partial Y'} + \mu \left(\frac{\partial^2 V'}{\partial X'^2} + \frac{\partial^2 V'}{\partial Y'^2}\right) = 0 \qquad (3.4)$$

$$\frac{\partial U'}{\partial X'} + \frac{\partial V'}{\partial Y'} = 0 \qquad (3.5)$$

where U', V' are the velocity components of the fluid, μ is viscosity and P' is the pressure. Taking into account the periodic nature of the longitudinal motion of the channel walls, the boundary conditions are

$$U'(X', t') = G'(X' - ct' + V_p' t') = c_1 \sin \frac{2\pi}{\lambda} (X' - ct' + V_p' t'),$$
$$\qquad (3.6)$$

$$V'(X', t') = \pm \frac{\partial H'}{\partial t} \quad ; \quad Y' = \pm H'(X', t')$$

and
$$U'(X', t') = -V_p',$$
$$V'(X', t') = \pm \frac{\partial H_1'}{\partial t} \quad ; \quad Y' = \pm H_1'(X', t') \qquad (3.7)$$

Under the present symmetric situation, the force equilibrium condition on the surface of the micro-organism can be written as

$$\int_S \tau' \, dS = \delta' \, \Delta P' \qquad (3.8)$$

where τ' is the resultant of the forces acting on the surface of the micro-organism $\Delta P'$ is the pressure rise over a wavelength and S is the surface of the micro-organism.

In a frame moving with velocity $c-V'_p$ in the positive axial direction, the boundaries of the channel and the micro-organism appear stationary. Using the following relations to transform various quantities from the stationary frame (X', Y', t') to the corresponding quantities in the moving frame (x, y, t) in the dimensionless form,

$$x = (X' - ct' + V'_p t')/\lambda, \quad y = Y'/a, \quad t = \frac{c}{\lambda}t'$$

$$u = (U' - c - V'_p)/c, \quad v = V'/\frac{ac}{\lambda}, \quad p = P'/\frac{\mu c \lambda}{a^2} \qquad (3.9)$$

$$(g, c_1, V_p) = (G', c'_1, V'_p)/c, \quad (h, h_1, \delta, \varepsilon, \varepsilon_1) = (h', h'_1, \delta', b', b'_1)/a$$

eqns. (3.3) - (3.5) are transformed as,

$$-\frac{\partial p}{\partial x} + \beta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad (3.10)$$

$$-\frac{\partial p}{\partial x} + \beta^2 (\beta^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) = 0 \qquad (3.11)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (3.12)$$

where $\beta = a/\lambda$.

For large wavelength $(\beta \ll 1)$ we can neglect the terms involving β^2 and higher orders and thus the eqns. (3.10) and (3.11) are simplified to,

$$-\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad (3.13)$$

$$- \frac{\partial p}{\partial y} = 0 \tag{3.14}$$

The boundary conditions for u are obtained from (3.6) and (3.7) as

$$u = V_p + g(x) - 1 \quad ; \quad y = \pm h(x)$$

$$u = -1 \qquad\qquad ; \quad y = \pm h_1(x) \tag{3.15}$$

where

$$h(x) = 1 + \varepsilon \sin 2\pi x$$

$$h_1(x) = \delta + \varepsilon_1 \sin 2\pi x \tag{3.16}$$

$$g(x) = c_1 \sin 2\pi x$$

Using the stress - strain relationship and neglecting terms containing β^2 and higher order, we obtain from (3.8)

$$\int_0^1 p \, dx = 0 \tag{3.17}$$

and

$$\int_0^1 \left[\left(\frac{\partial u}{\partial y}\right) y = h_1(x) + \frac{dh_1(x)}{dx} p \right] dx = \delta \Delta p \tag{3.18}$$

Using the symmetry we solve these equations for $y \geq 0$ region only.

3.2.2 ANALYSIS :

The equation (3.13) can be solved, using boundary conditions (3.15), as

$$u = \frac{1}{2} \left(\frac{\partial p}{\partial x}\right) \left[y^2 - (h+h_1)y + hh_1\right] + (g+V_p) \frac{(y-h_1)}{h-h_1} - 1 \tag{3.19}$$

The volumetric rate of flow is

$$q = \int_{h_1}^{h} u \, dy = -\frac{1}{12}\left(\frac{\partial p}{\partial x}\right)(h-h_1)^3 + \frac{1}{2}(g + V_p - 2)(h-h_1) \tag{3.20}$$

The expression (3.20) can be rewritten as

$$\frac{\partial p}{\partial x} = -12\frac{q}{(h-h_1)^3} + \frac{6(g+V_p-2)}{(h-h_1)^2} \tag{3.21}$$

Integrating equation of continuity (3.12), it can be seen that the flux q is constant in the wave-frame. Integrating equation (3.21) over a wavelength we get

$$\Delta p = -12 I_{11}q + 6 I_{12} V_p + 6 I_{13} - 12 I_{12} \tag{3.22}$$

where $I_{11} = \int_{o}^{1} \frac{dx}{(h-h_1)^3}$, $I_{12} = \int_{o}^{1} \frac{dx}{(h-h_1)^2}$, $I_{13} = \int_{o}^{1} \frac{g(x)}{(h-h_1)^2} \, dx$

$$\tag{3.23}$$

Using expressions for u and $\frac{\partial p}{\partial x}$ from (3.19) and (3.21) the force equilibrium condition (3.18) can be written as

$$0 = 6 I_{21}q - 2 I_{22} V_p - 2 I_{23} + 6 I_{24} \tag{3.24}$$

where $I_{21} = \int_{o}^{1} \frac{h+h_1}{(h-h_1)^3} dx$, $I_{22} = \int_{o}^{1} \frac{h+2h_1}{(h-h_1)^2}$

$$\tag{3.25}$$

$$I_{23} = \int_{o}^{1} \frac{(h+2h_1)g(x)}{(h-h_1)^2} \, dx , \quad I_{24} = \int_{o}^{1} \frac{h+h_1}{(h-h_1)^2}$$

The equations (3.22) and (3.24) are linear equations in V_p and q and can be solved as

$$V_p = \frac{1}{M_{oo}} (M_{11} \Delta p + M_{12}) \tag{3.26}$$

$$q = \frac{1}{M_{oo}} - (M_{21} \, \Delta p + M_{22}) \qquad (3.27)$$

with $M_{oo} = 6I_{12} \cdot I_{21} - 4I_{22} \cdot I_{11}$

$M_{11} = I_{21}$

$M_{12} = -6I_{21} \cdot I_{13} + 4I_{11} \cdot I_{23} + 12I_{12} \cdot I_{21} - 12I_{11} \cdot I_{24}$

$M_{21} = \frac{1}{3} I_{22} \qquad (3.28)$

$M_{22} = 2I_{12}(I_{23} - 3I_{24}) + 2I_{22}(2I_{12} - I_{13})$

The fluxes in stationary and wave frames are related as

$$Q = q - (V_p - 1)(H - H_1) \qquad (3.29)$$

The time - averaged flux in stationary frame $\bar{Q}$, therefore
is related with q and V_p as

$$\bar{Q} = q - (V_p - 1)(1 - \delta) \qquad (3.30)$$

The integrals (3.23) and (3.25) can easily be evaluated
in terms of the integrals given in appendix 2(A) (Chapter II)
and using eqns. (3.26) - (3.30) V_p and $\bar{Q}$ can be obtained.

3.2.3 RESULTS AND DISCUSSIONS :

The expressions (3.26) - (3.30) are computed for
various values of $c_1, \varepsilon, \Delta p, \varepsilon_1, \delta$ and the effects of various
parameters on V_p and $\bar{Q}$ are shown in fig. 3.3 - 3.11. Effects
of $c_1, \varepsilon \, \Delta p$ on V_p as function of ε_1 are shown in figs. 3.3 - 3.7.
It can be noticed from these figures that

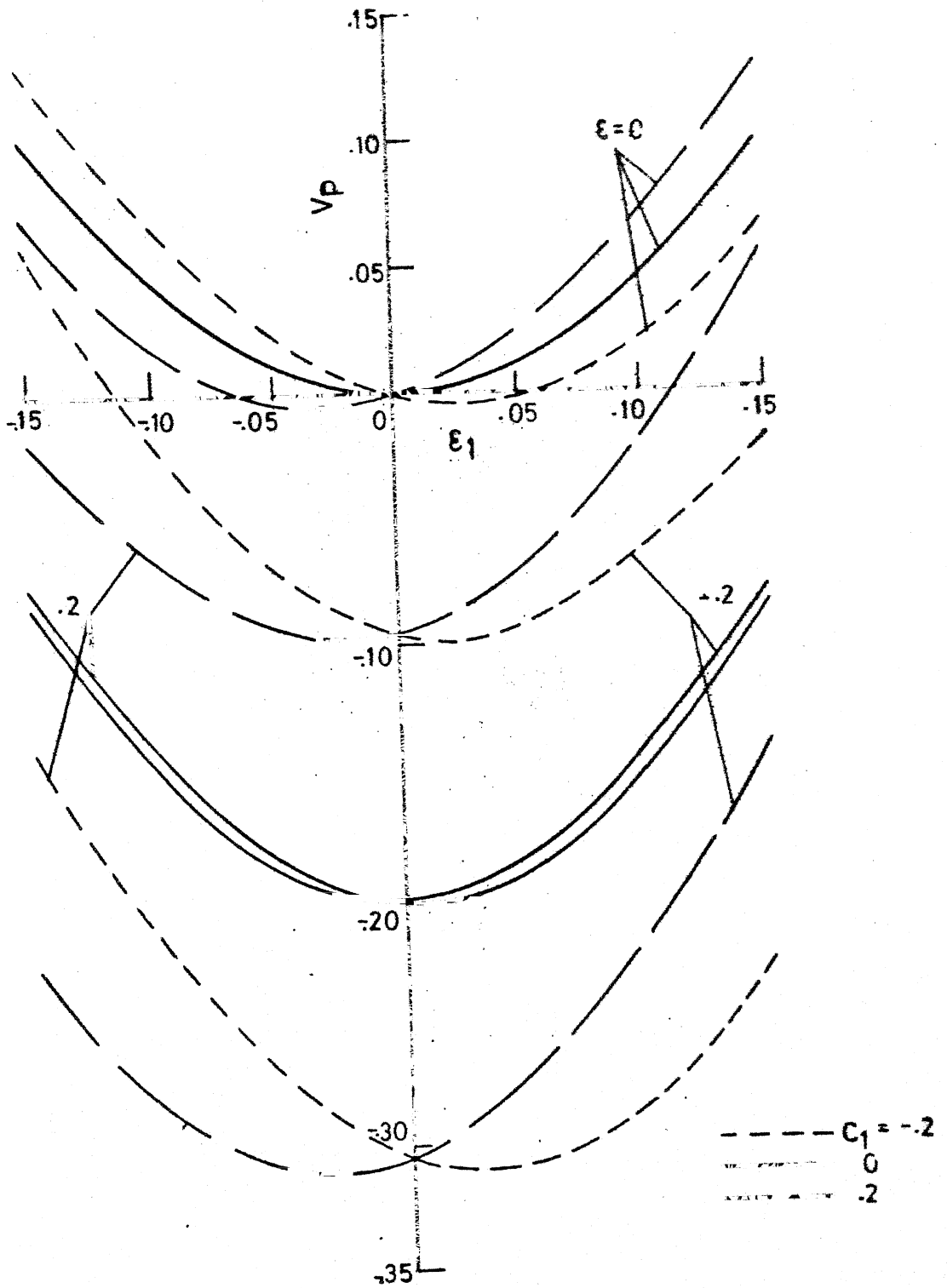$$V_p(c_1, \varepsilon_1, \varepsilon) = V_p(-c_1, -\varepsilon_1, -\varepsilon)$$

FIG. 3.3 EFFECT OF ε AND C_1 ON V_P (CHANNEL) ($\Delta p = 0$, $\delta = 0.2$)
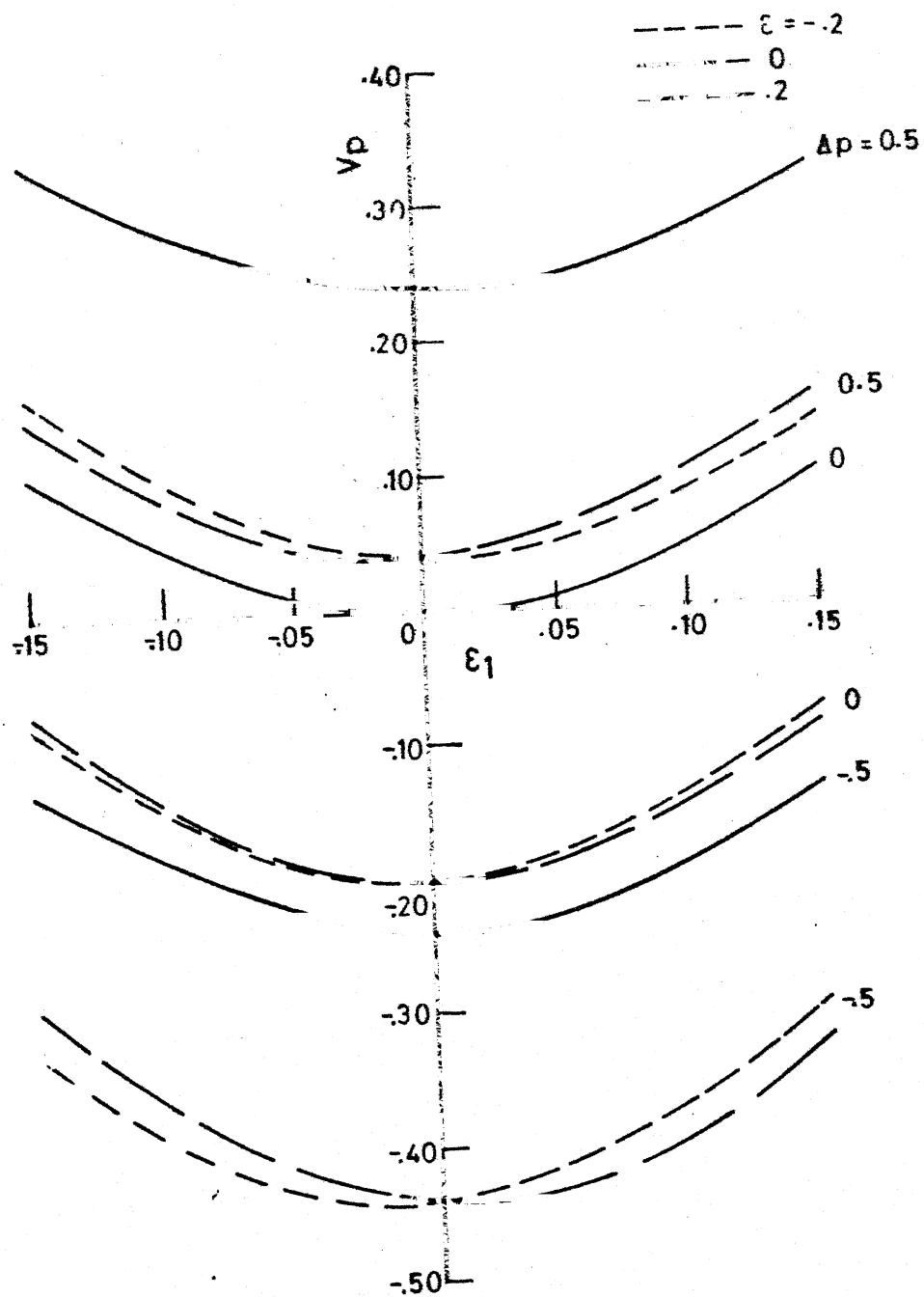
FIG. 3.4   EFFECT OF Δp AND ε ON V_p ($C_1 = 0, \delta = 0.2$) (CHANNEL)
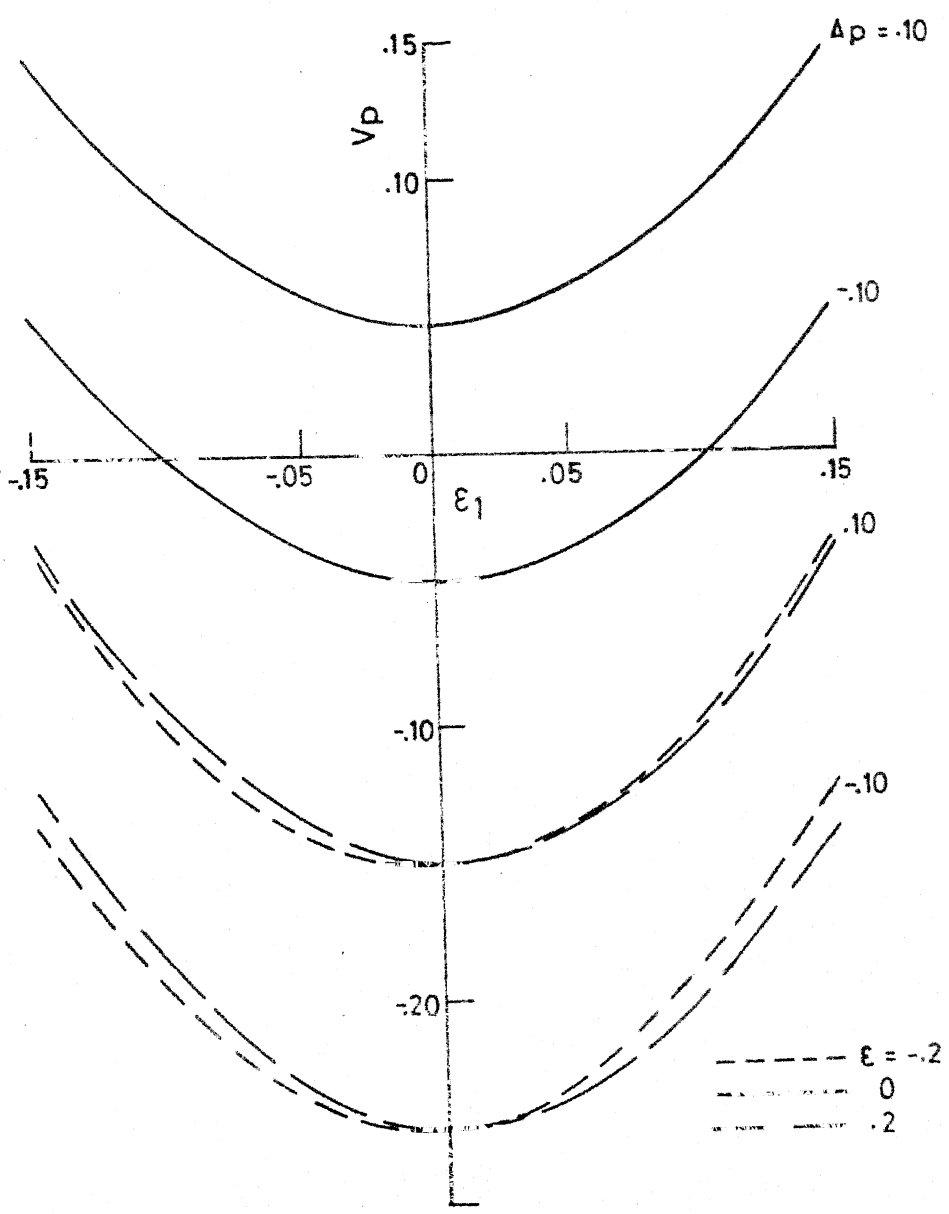
FIG. 3·5   EFFECT OF Δp AND $\mathcal{E}$ ON V_p (CHANNEL) ($C_1=0, \delta=0.2$)
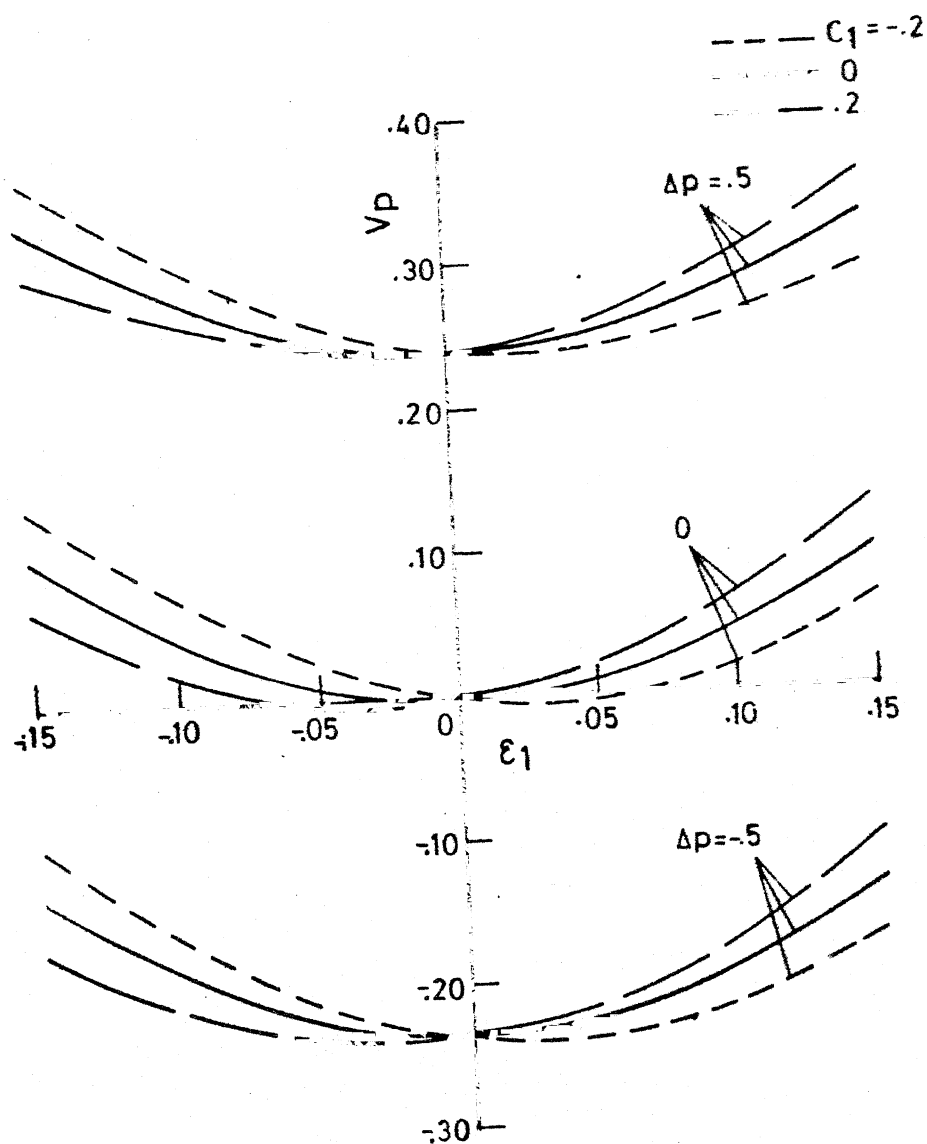
FIG. 3.6   EFFECT OF Δp AND C_1 ON V_p ($\varepsilon = 0$, $\delta = 0.2$)(CHANNEL)
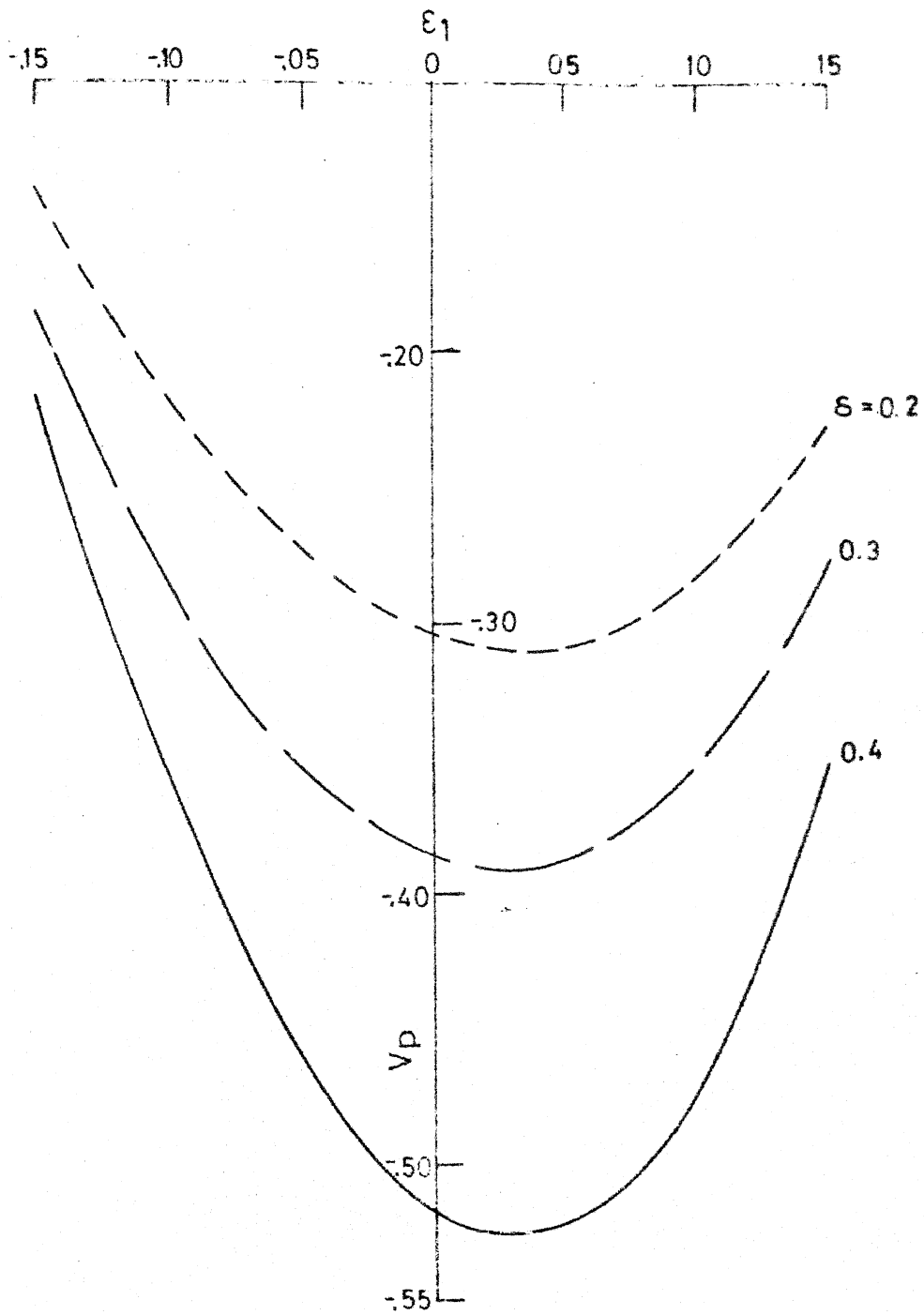
FIG 3.7   EFFECT OF δ ON V_p (CHANNEL) ($\Delta p = 0, \mathcal{E} = 0, C_1 = -.2$)
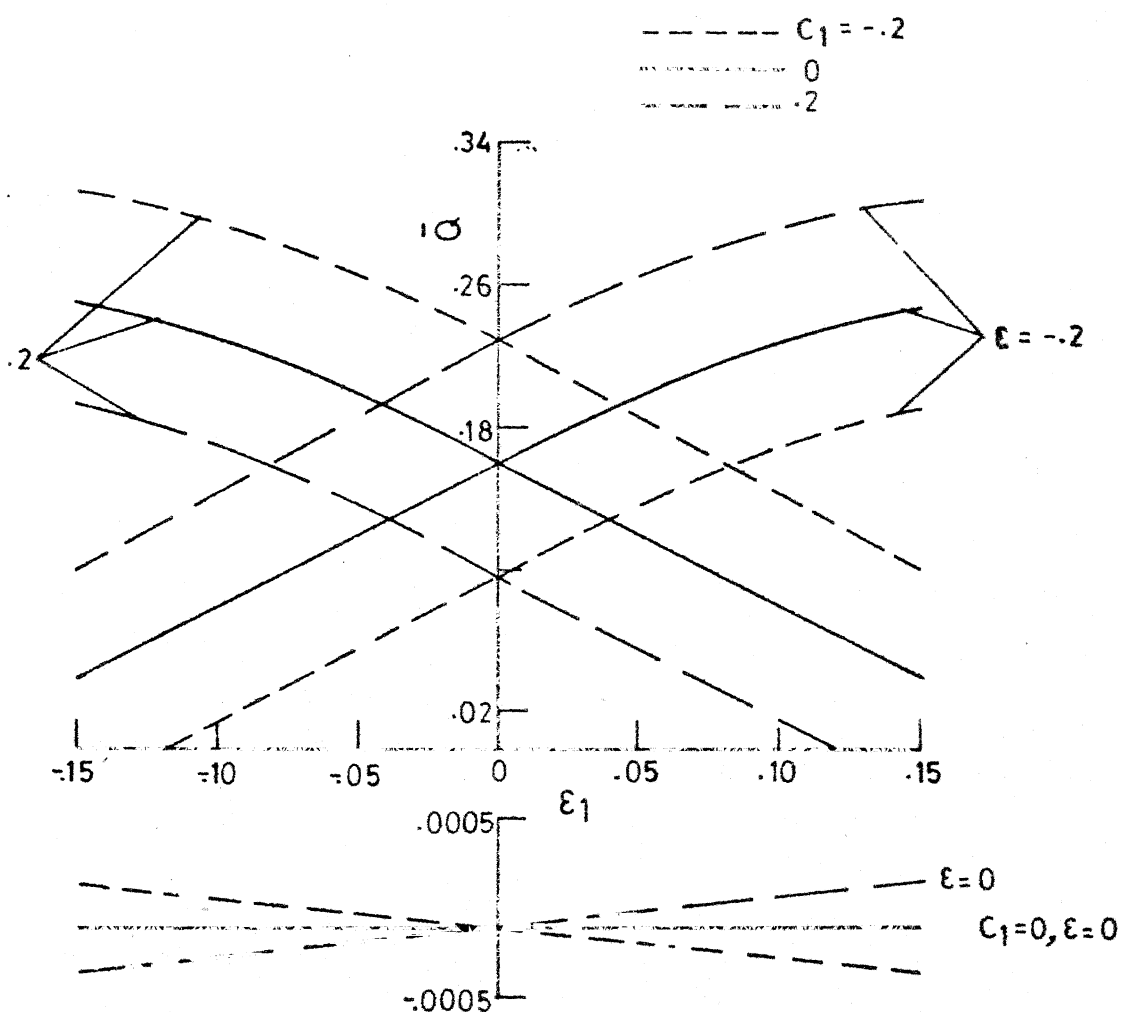
FIG. 3.8 EFFECT OF C_1 AND ε ON $\bar{Q}$ (CHANNEL) ($\Delta p = 0$, $\delta = 0.2$)
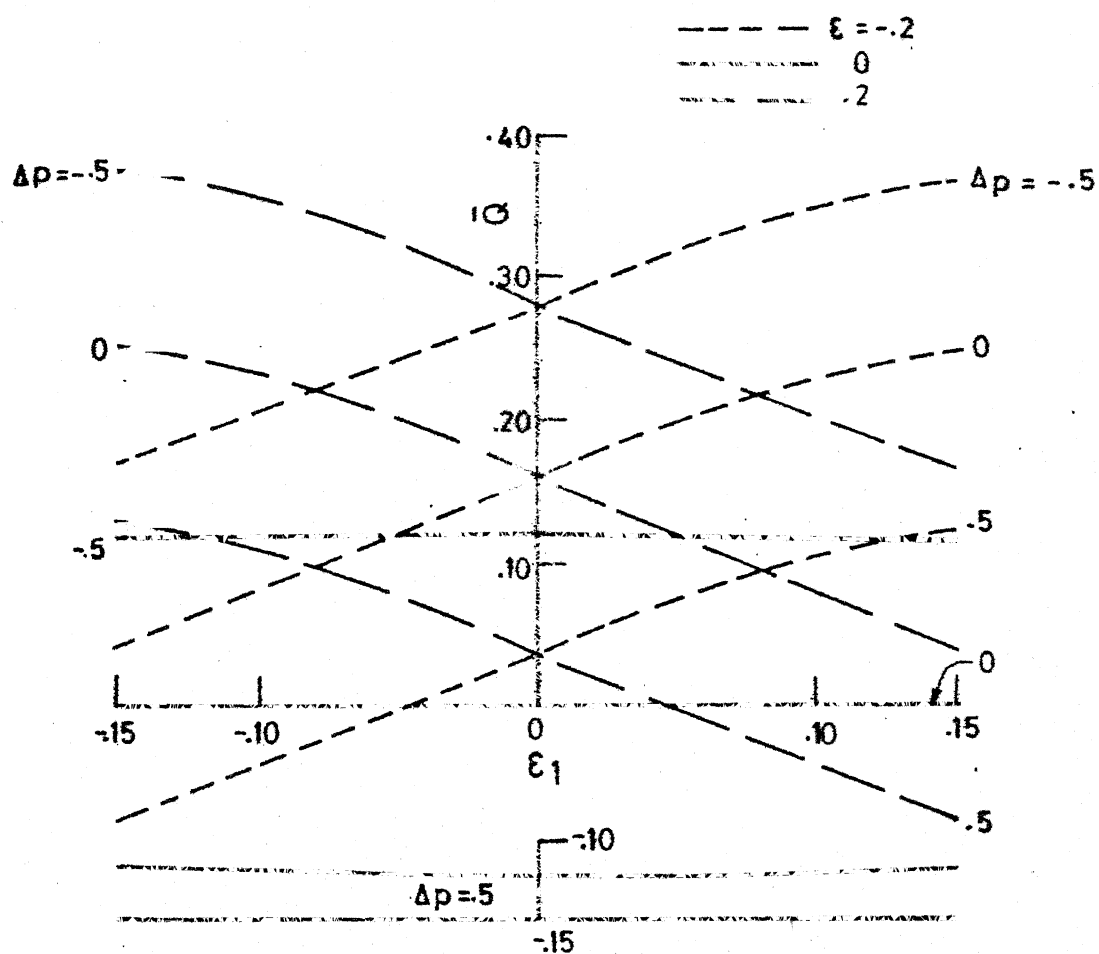
FIG. 3·9 EFFECT OF Δp AND ε ON $\bar{Q}$ (CHANNEL) ($C_1 = 0, \delta = 0.2$)
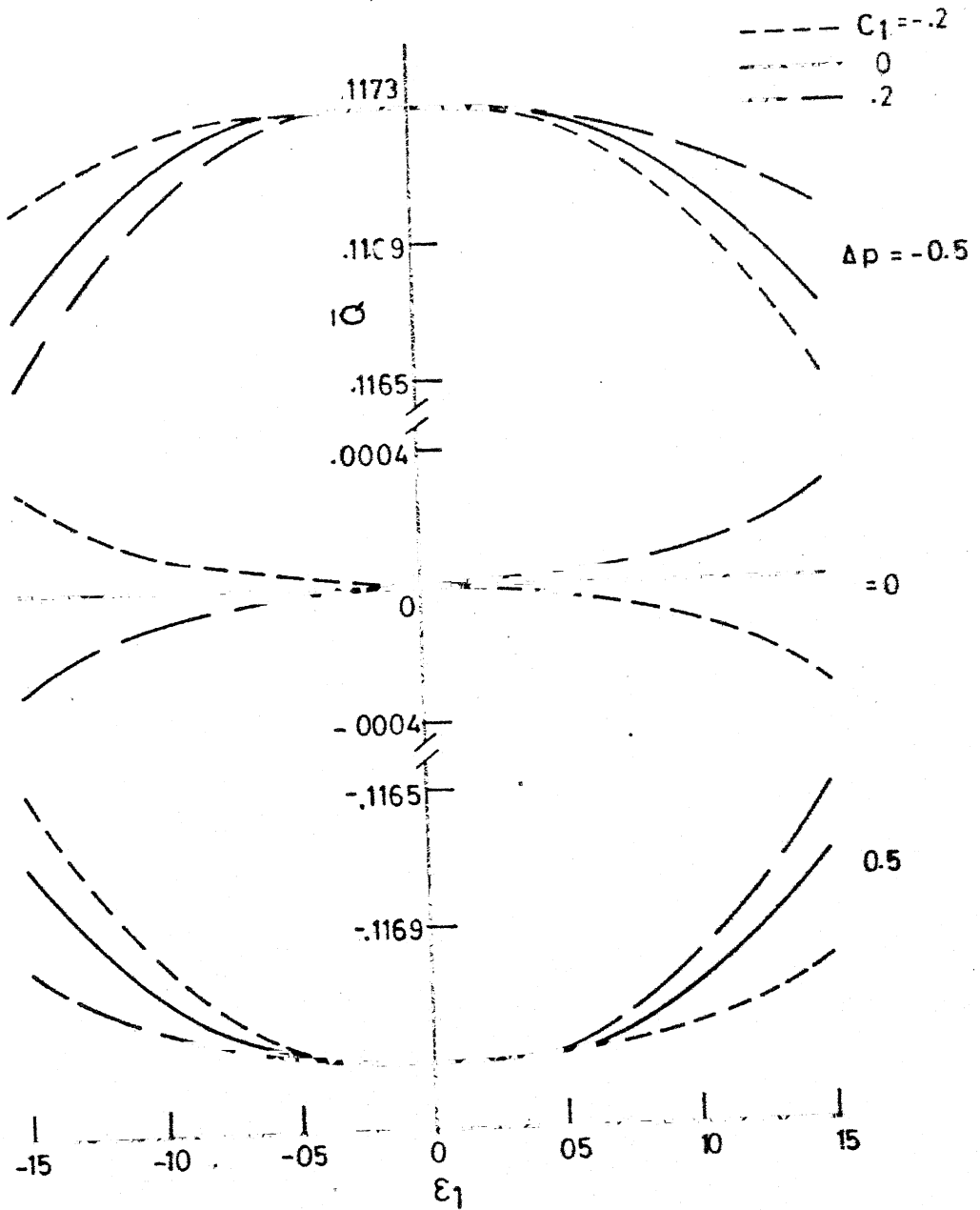
G. 3.10   EFFECT OF Δp AND C_1 ON $\bar{Q}$ (CHANNEL) ($\mathcal{E} = 0, \delta = 0.2$)
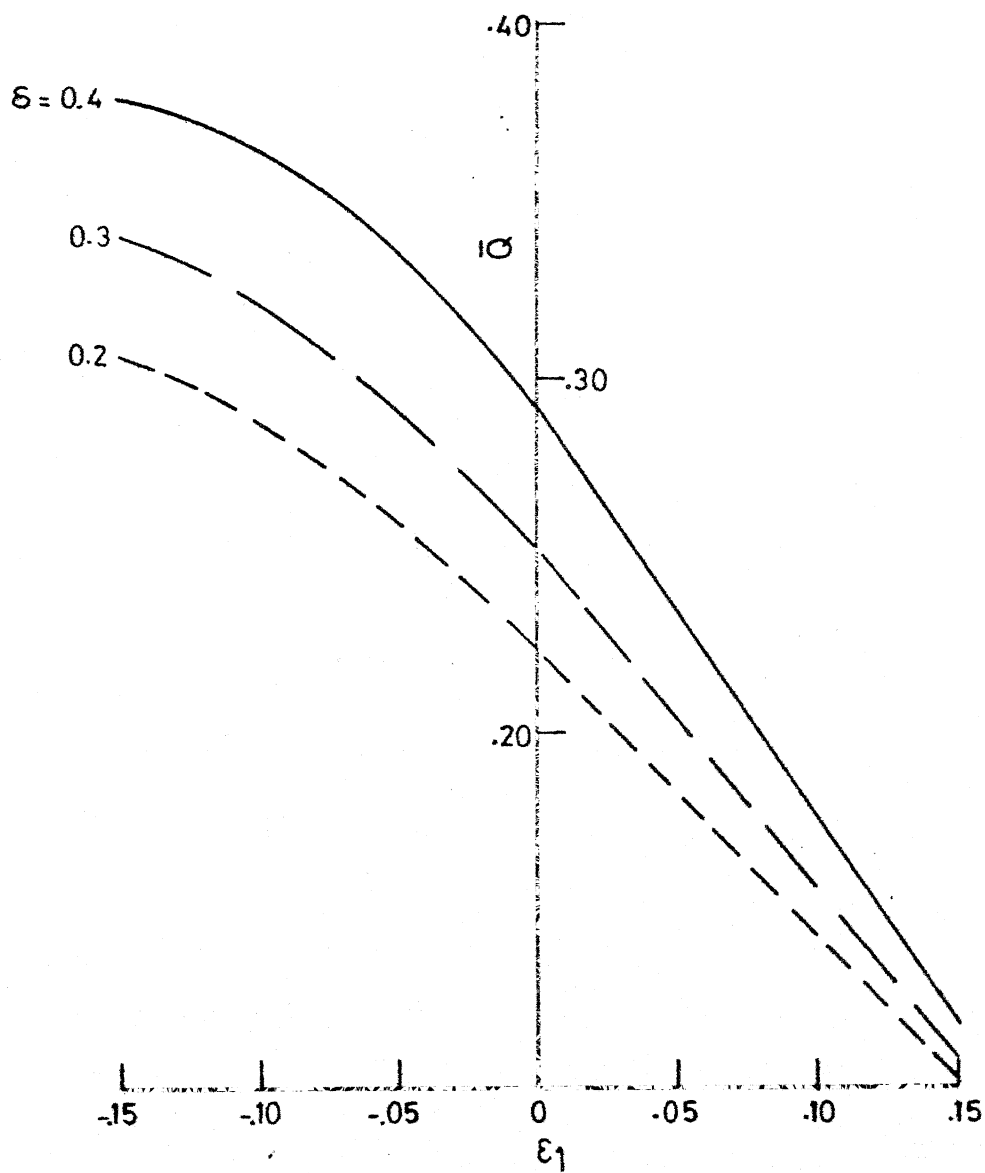
FIG. 3.11   EFFECT   OF δ ON $\bar{Q}$ ($\Delta p = 0, \mathcal{E} = 0.2, C_1 = -2$)   (CHANNEL)

and that V_p increases as magnitude of ε_1 increases. For fixed value of c_1, V_p decreases as ε increases and this effect is further enhanced for positive values of c_1. (fig. 3.3).
Effect of Δp is to increase V_p and in case of pressure drop (i.e. $\Delta p < 0$) negative values of V_p are obtained (fig.3,4,3.5).
Similar effects of Δp and c_1 can be noticed from fig. 3.6.
Effect of δ on V_p is shown in fig. 3.7 and it can be observed that an increase in δ causes a decrease in V_p.

Effect of various parameters on time average flux $\bar{Q}$ is graphically shown in figs. 3.8 – 3.10. In all the cases the effects of various parameters on $\bar{Q}$ are opposite to those on V_p. It is noticed that

$$\bar{Q}(c_1, \varepsilon_1, \varepsilon) = \bar{Q}(-c_1, -\varepsilon_1, -\varepsilon)$$

It can be seen that $\bar{Q}$ increases with ε_1 when $\varepsilon < 0$, a reverse trend is, however, noticed for the case $\varepsilon > 0$.
A similar effect of c_1 is also observed. For fixed values of c_1 and ε_1, increase in $\bar{Q}$ with increase in ε is noticed.
The effect of increasing Δp is to decrease $\bar{Q}$. In the particular case when $\varepsilon = 0$, $\bar{Q}$ decreases as magnitude of ε_1 increase for $\Delta p < 0$ and an opposite trend is noticed for $\Delta p > 0$ (fig. 3.9).

3.3 TRANSPORT THROUGH A TUBE

3.3.1 MATHEMATICAL MODEL (CASE 2)

Let us consider the flow of an incompressible viscous fluid in a cylindrical tube of radius a, where the central

core region (Fig. 3.12) of radius δ' is moving with a velocity V'_p to be determined, in the negative axial direction. The fluid flow is confined only to the region between the boundaries of core and the tube. Let a peristaltic wave of wavelength λ and amplitude b' pass over the wall of the tube with a speed $c-V'_p$ and let another peristaltic wave of amplitude b'_1 pass over the boundary of the core region. It is assumed that in the steady state, this peristaltic wave tries to align itself to the wave passing over the wall of the tube and hence acquires the same speed and wavelength. The shape of the wall of tube H' and the inner core region H'_1 are, therefore at the instant t', given by

$$H'(X',t') = a + b' \sin \frac{2\pi}{\lambda}(X' - ct' + V'_p t) \qquad (3.31)$$

$$H'_1(X',t') = \delta' + b'_1 \sin \frac{2\pi}{\lambda}(X' - ct' + V'_p t') \qquad (3.32)$$

The Reynolds number involved here is of order 10^{-3} as in the section 3.2; the equations of motion therefore are

$$-\frac{\partial P'}{\partial X'} + \mu\left[\frac{\partial^2 U'}{\partial X'^2} + \frac{1\partial}{R'\partial R'}(R'\frac{\partial U'}{\partial R'})\right] = 0 \qquad (3.33)$$

$$-\frac{\partial P'}{\partial R'} + \mu\left[\frac{\partial^2 V'}{\partial X'^2} + \frac{1\partial}{R'\partial R'}(R'\frac{\partial V'}{\partial R'})\right] = 0 \qquad (3.34)$$

$$\frac{\partial U'}{\partial X'} + \frac{\partial V'}{\partial R'} + \frac{V'}{R'} = 0 \qquad (3.35)$$

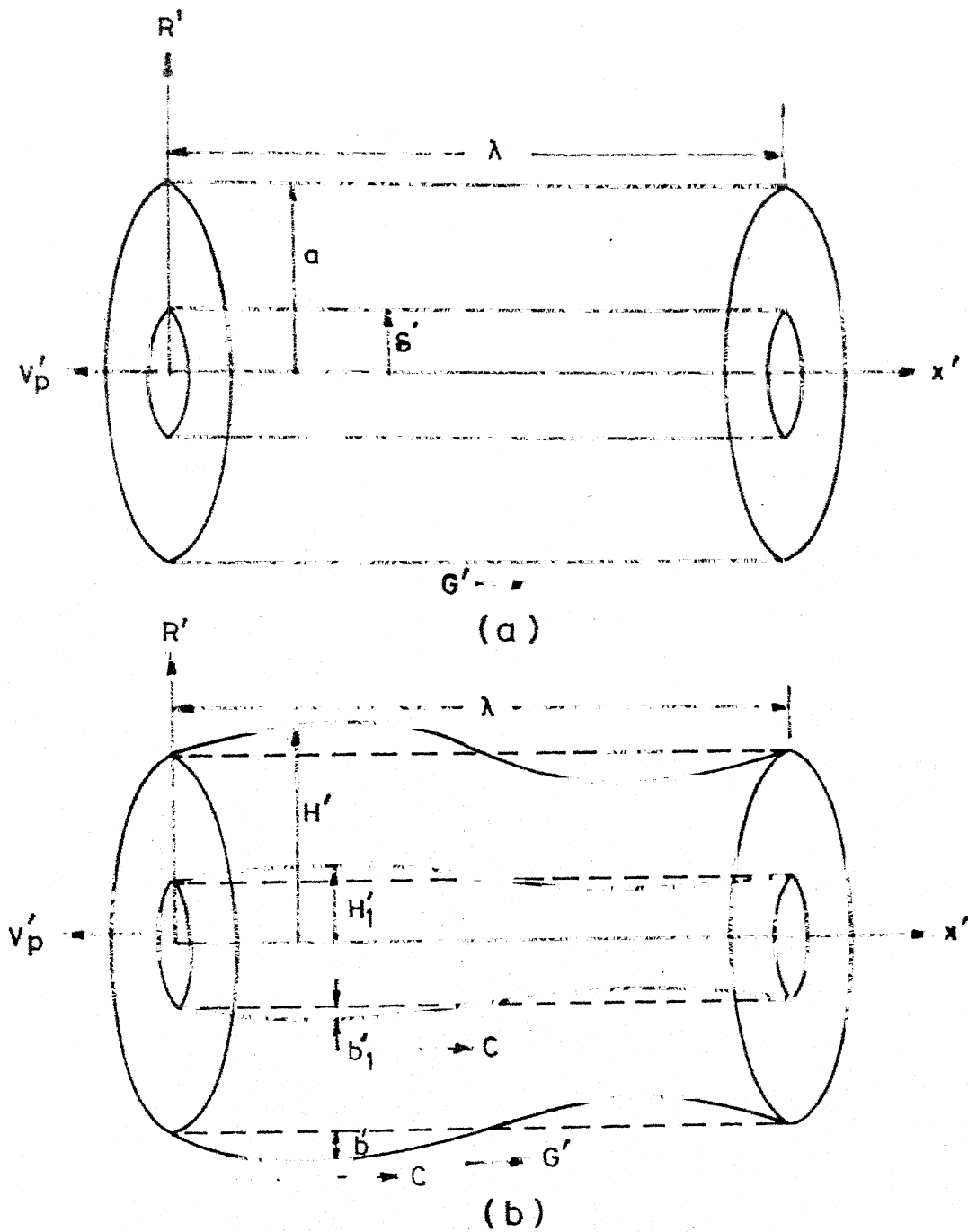FIG. 3.12 MOTION OF A MICRO ORGANISM THROUGH A TUBE
(a) Without peristalsis (b) With peristalsis

The boundaries conditions are (as in section 3.2)

$$U' = G'(X', t') \equiv c_1' \sin \frac{2\pi}{\lambda} (X' - ct' + V_p' t')$$

$$V' = \frac{\partial H'}{\partial t} \qquad\qquad\qquad\qquad \text{at } R' = H'(X', t')$$

(3.36)

and

$$U' = -V_p'$$

$$V' = \frac{\partial H_1'}{\partial t} \qquad\qquad \text{at} \qquad R' = H_1'(X', t') \qquad (3.37)$$

The force equilibrium condition on the self propelling micro-organism is

$$\int_S \tau' \, dS = \pi \delta'^2 \Delta P' \qquad\qquad (3.38)$$

In wave frame moving with velocity $c - V_p'$ in the positive axial direction, we define the following non-dimensionalized variables

$$x = \frac{X' - ct' + V_p' t'}{\lambda}, \quad r = \frac{R'}{a}, \quad t = \frac{t' c}{\lambda}, \quad p = \frac{P' a^2}{\mu c \lambda}$$

$$u = \frac{U' - c + V_p'}{c}, \quad v = \frac{V' \lambda}{ca}, \quad (h, h_1, \varepsilon, \varepsilon_1, \delta) = (H', H_1', b', b_1', \delta')/a$$

$$(g, c_1, V_p) = (g', c_1', V_p')/c.$$

(3.39)

The equations (3.33)-(3.35) become

$$-\frac{\partial p}{\partial x} + \beta^2 \frac{\partial^2 u}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) = 0 \qquad (3.40)$$

$$-\frac{\partial p}{\partial r} + \beta^2 \left[\beta^2 \frac{\partial^2 v}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{\partial v}{\partial r} \right) \right] = 0 \qquad (3.41)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0 \qquad\qquad (3.42)$$

where $\beta = a/\lambda$.

For long wavelength approximation $\beta \ll 1$. The equations (3.40) and (3.41), are therefore simplifed as,

$$-\frac{\partial p}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r}) = 0 \qquad\qquad (3.43)$$

$$\frac{\partial p}{\partial r} = 0 \qquad\qquad (3.44)$$

We get the corresponding boundary conditions for u as

$$u = g(x) + V_p - 1 \qquad at\ r = h(x)$$
$$u = -1 \qquad\qquad at\ r = h_1(x) \qquad\qquad (3.45)$$

where

$$g(x) = c_1 \sin 2\pi x$$

$$h(x) = 1 + \varepsilon \sin 2\pi x \qquad\qquad (3.46)$$

$$h_1(x) = \delta + \varepsilon_1 \sin 2\pi x$$

The force equilibrium condition (3.38) becomes

$$2\pi \int_0^1 [(r\frac{\partial u}{\partial r})_{r=h_1} + \frac{dh_1(x)}{dx} h_1(x)p]dx = \pi \delta^2 \Delta p \qquad\qquad (3.47)$$

and $\int_0^1 p\ dx = 0$

where Δp is the pressure rise over a wavelength.

3.3.2 ANALYSIS

The equation (3.43) can be solved, using boundary conditions (3.45) as

$$u = -1 + \frac{1}{4} \frac{\partial p}{\partial x} \left[r^2 + \frac{(h_1^2 - h^2)\ln(r/h_1)}{\ln(h/h_1)} - h_1^2 \right] + \frac{(g(x)+V_p)\ln(r/h_1)}{\ln(h/h_1)}$$

$$(3.48)$$

The flux in the wave frame, $q (= Q'/\pi a^2 c)$ is obtained as

$$q = 2 \int_{h_1}^{h} r \, u \, dr = \frac{1}{8} F_{11} \frac{\partial p}{\partial x} + F_{12} + (g(x)+V_p)F_{13}$$

which can be rewritten as

$$-\frac{\partial p}{\partial x} = \frac{8q}{F_{11}} - 8 \frac{F_{12}}{F_{11}} - 8(g(x) + V_p) \frac{F_{13}}{F_{11}} \qquad (3.49)$$

where

$$F_{11}(x) = (h_1^4 - h^4) + \frac{(h_1^2 - h^2)^2}{\ln(h/h_1)}$$

$$(3.50)$$

$$F_{12}(x) = h_1^2 - h^2$$

$$F_{13}(x) = h^2 + \frac{h_1^2 - h^2}{2\ln(h/h_1)}$$

Integrating (3.49) over a wavelength we get

$$\Delta p = 8I_{11}q - 8I_{12}V_p - 8c_1 I_{13} - 8I_{14} \qquad (3.51)$$

with $\quad I_{11} = \int_0^1 \frac{dx}{F_{11}(x)} \; ; \quad I_{12} = \int_0^1 \frac{F_{13}(x)}{F_{11}(x)} \, dx$

$$(3.52)$$

$$I_{13} = \int_0^1 \frac{F_{13}(x)}{F_{11}(x)} \sin 2\pi x \, dx ; \quad I_{14} = \int_0^1 \frac{F_{12}(x)}{F_{11}(x)} \, dx$$

Equation (3.51) provides a linear relation in V_p and q. Another relation is obtained from the force equilibrium condition (3.47) which, after substituting expressions for

u and $\frac{\partial p}{\partial x}$, from eqns. (3.48), (3.49), become

$$o = 4I_{21}q - 2I_{22}V_p - 2I_{23}c_1 - 4I_{24} \qquad (3.53)$$

where $\quad I_{21} = \int\limits_{o}^{1} \frac{dx}{F_{21}(x)} \qquad ; \; I_{22} = \int\limits_{o}^{1} \frac{F_{22}(x)}{F_{21}(x)}$

$$(3.54)$$

$$I_{23} = \int\limits_{o}^{1} \frac{F_{22}(x)}{F_{21}(x)}\sin 2\pi x dx ; I_{24} = \int\limits_{o}^{1} \frac{F_{12}(x)}{F_{21}(x)}$$

and $\quad F_{21}(x) = (h_1^2 + h^2)\ln(h/h_1) + (h_1^2 - h^2)$

$$(3.55)$$

$$F_{22}(x) = 2\frac{F_{13}(x)}{F_{21}(x)} - \frac{1}{\ln(h/h_1)}$$

Solving equations (3.51) and (3.53) we get the expressions for V_p and q as

$$V_p = - \frac{M_{11}\Delta p + M_{12}c_1 + M_{13}}{M_{oo}} \qquad (3.56)$$

$$q = \frac{M_{21}\Delta p + M_{22}c_1 + M_{23}}{M_{oo}} \qquad (3.57)$$

where $\quad M_{oo} = 8(I_{22}I_{11} - 2I_{12}I_{21})$

$$M_{11} = 2I_{21}$$

$$M_{12} = 8(2I_{13}I_{21} - I_{23}I_{11})$$

$$M_{13} = 16(I_{14}I_{21} - I_{24}I_{11})$$

$$(3.58)$$

$$M_{21} = I_{22}$$

$$M_{22} = 8(I_{13}I_{22} - I_{23}I_{12})$$

$$M_{23} = 8(I_{14}I_{22} - 2I_{24}I_{12})$$

The time - averaged flux $\bar{Q}$ in stationary frame can be obtained by using following relation

$$\bar{Q} = q + (1-V_p)(1 - \dot{\lambda}^2 + \frac{\varepsilon^2}{2} - \frac{\varepsilon_1^2}{2}) \qquad (3.59)$$

3.33 RESULTS AND DISCUSSIONS

The integrals given by expressions (3.52) and (3.54) are computed numerically using trapezoidal rule and the expressions for V_p and $\bar{Q}$ (3.56, 3.59) are computed. It is observed that the qualitative behaviour of the results in this case is similar to those discussed in section 3.2.3 corresponding to flow through a channel.

In figs 3.13-3.18 , V_p as a function of ε_1 is plotted for various values of c_1, ε and Δp. V_p is shown to decrease with ε_1, attains a minimum and increase again ($\varepsilon = \pm .05$) (fig. 3.13). For higher value of ε ($\pm .2$) this minima is not seen in the observed range of ε_1 and V_p monotonically decreases ($\varepsilon = .2$) or increases ($\varepsilon = -.2$) (fig. 3.14). Comparing these two figures (3.13,3.14) it can be observed that range of V_p is increased as ε increases. The effect of increasing c_1 is to increase the magnitude of V_p and that of Δp is to decrease it. The maximum (or minimum negative) value of V_p is obtained when $\varepsilon < 0$ and $\Delta p > 0$. Effect of δ is also similar as observed for channel case for ($|\varepsilon| \leq .05$) (fig. 3.18) .
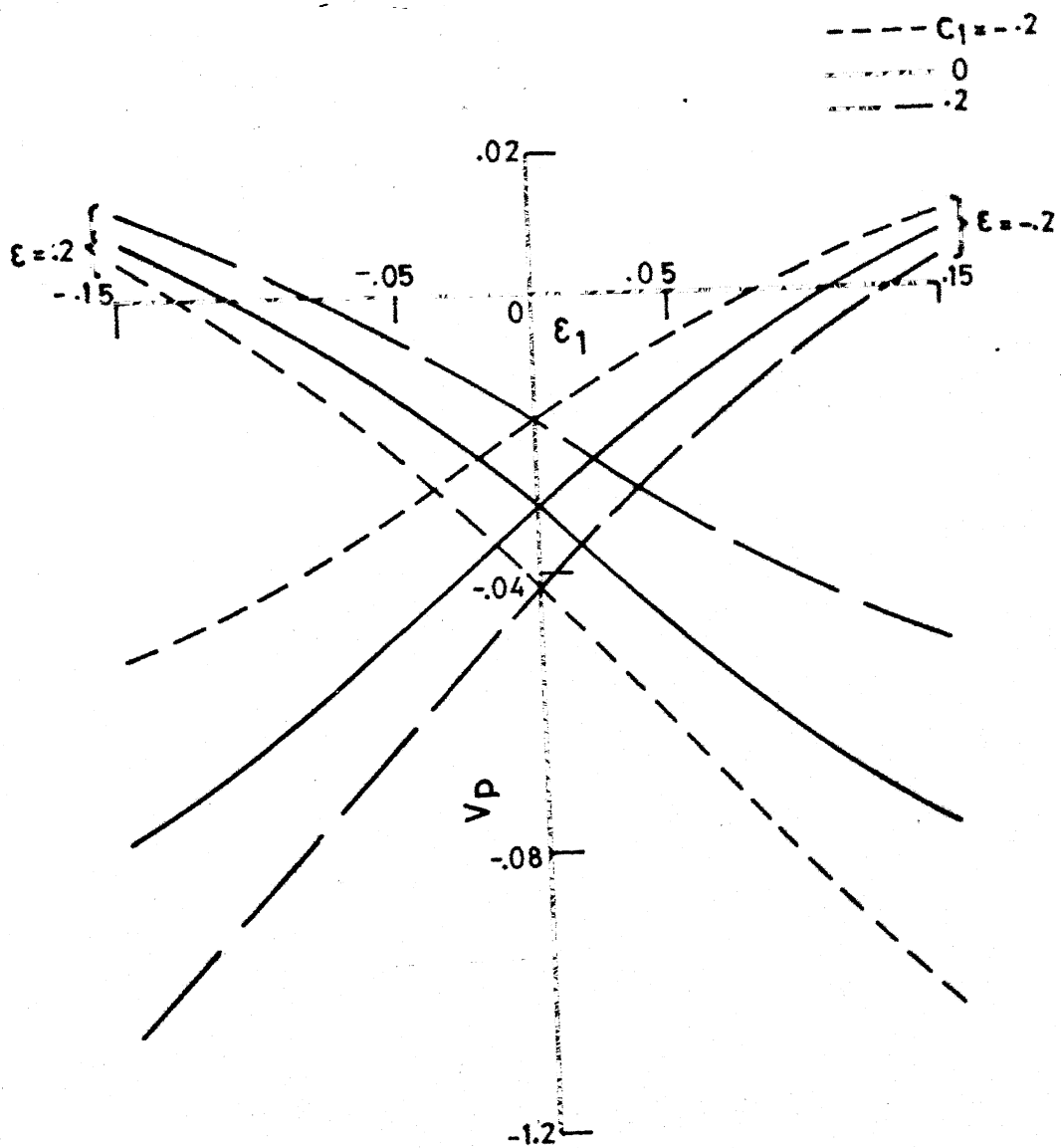
FIG. 3.13   EFFECT OF ε AND C_1 ON VARIATION OF V_P WITH ε_1 (TUF

$(\delta = 0.2 , \Delta p = 0.0)$
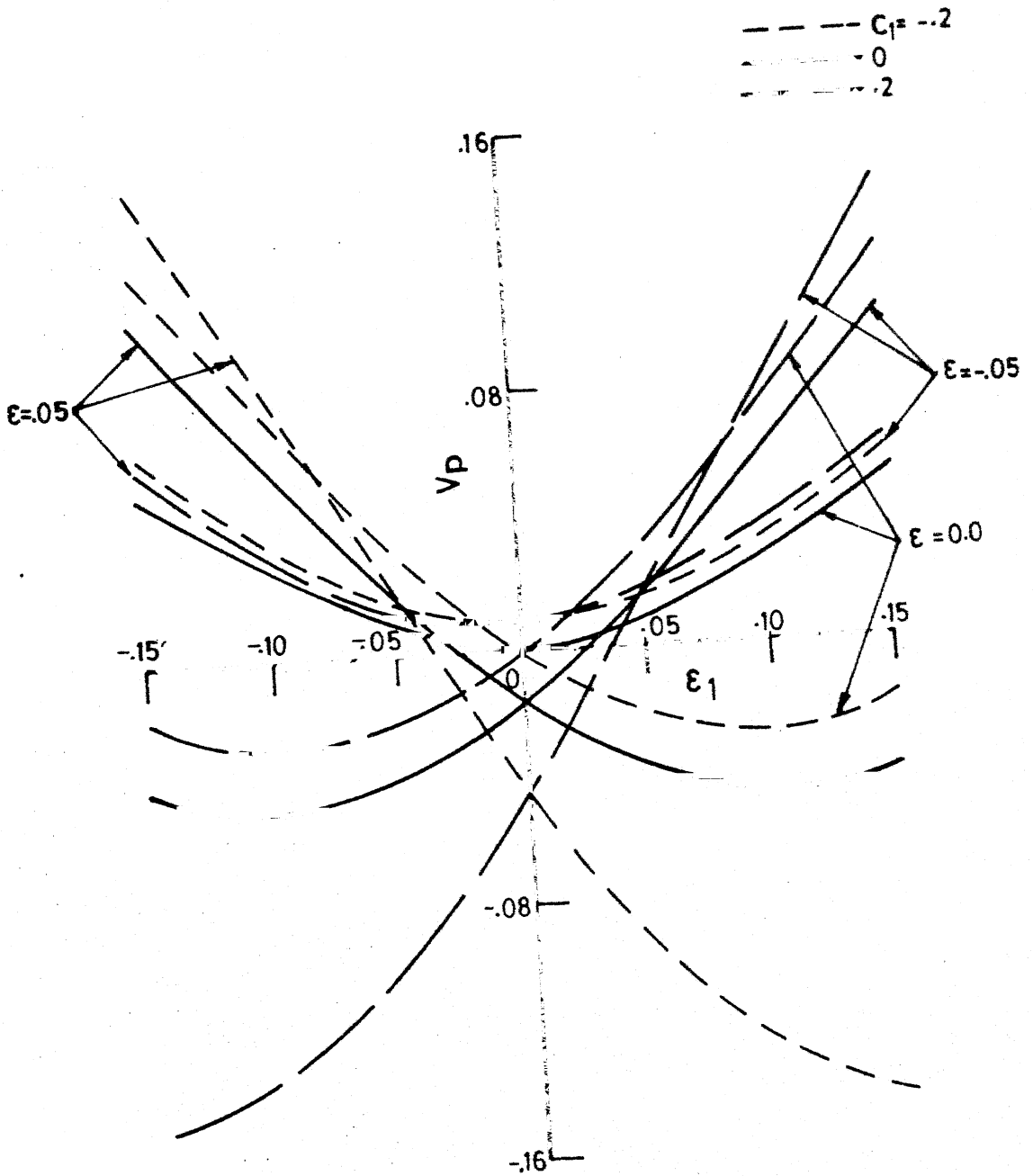
FIG. 3.14 EFFECT OF C_1 AND ε ON V_p (TUBE) ($\Delta p = 0, \delta = .2$)
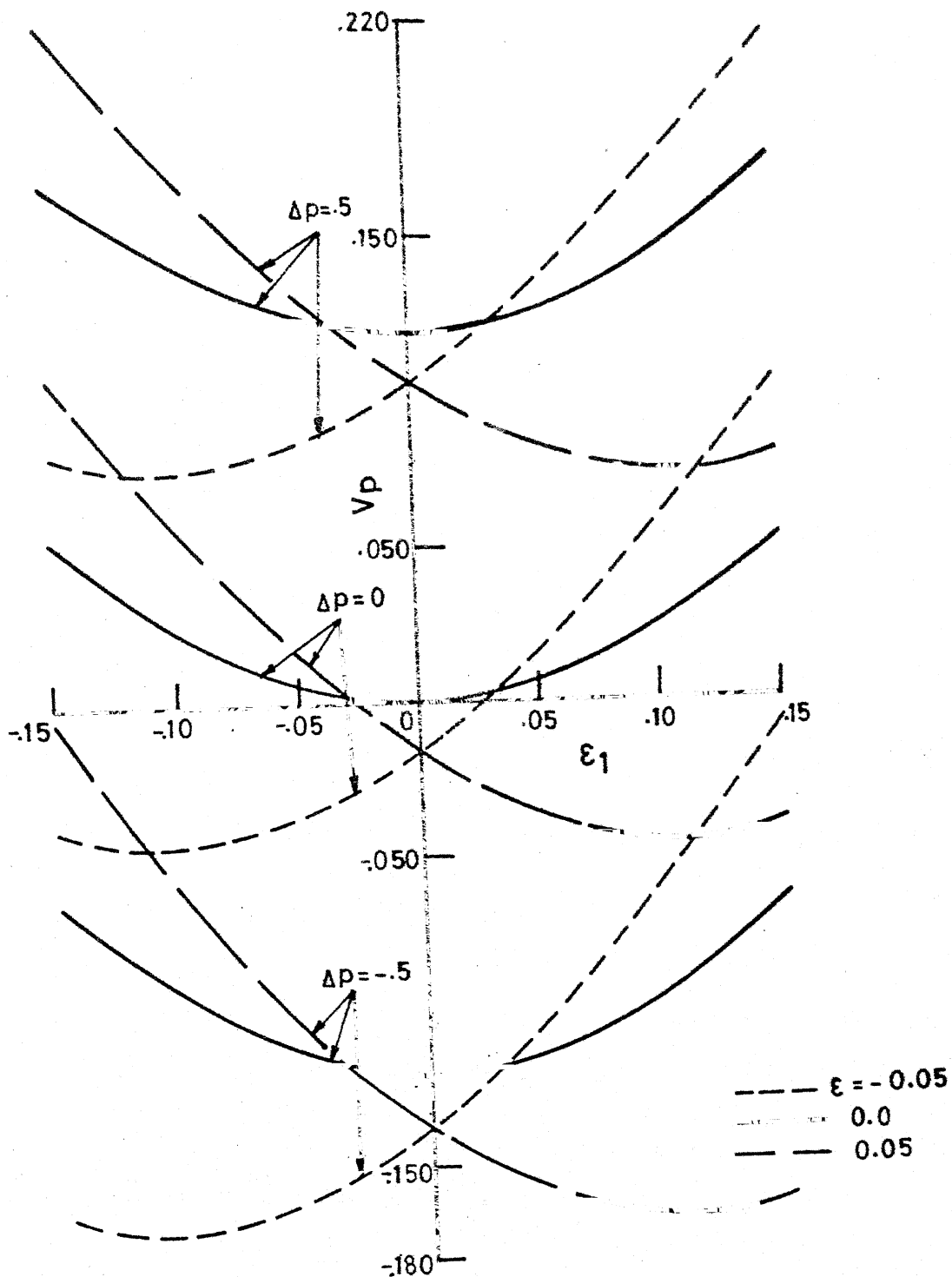
FIG. 3.15 EFFECT OF Δp AND ε ON V_p (TUBE)($\delta = 0.2$, $C_1 = 0.0$)
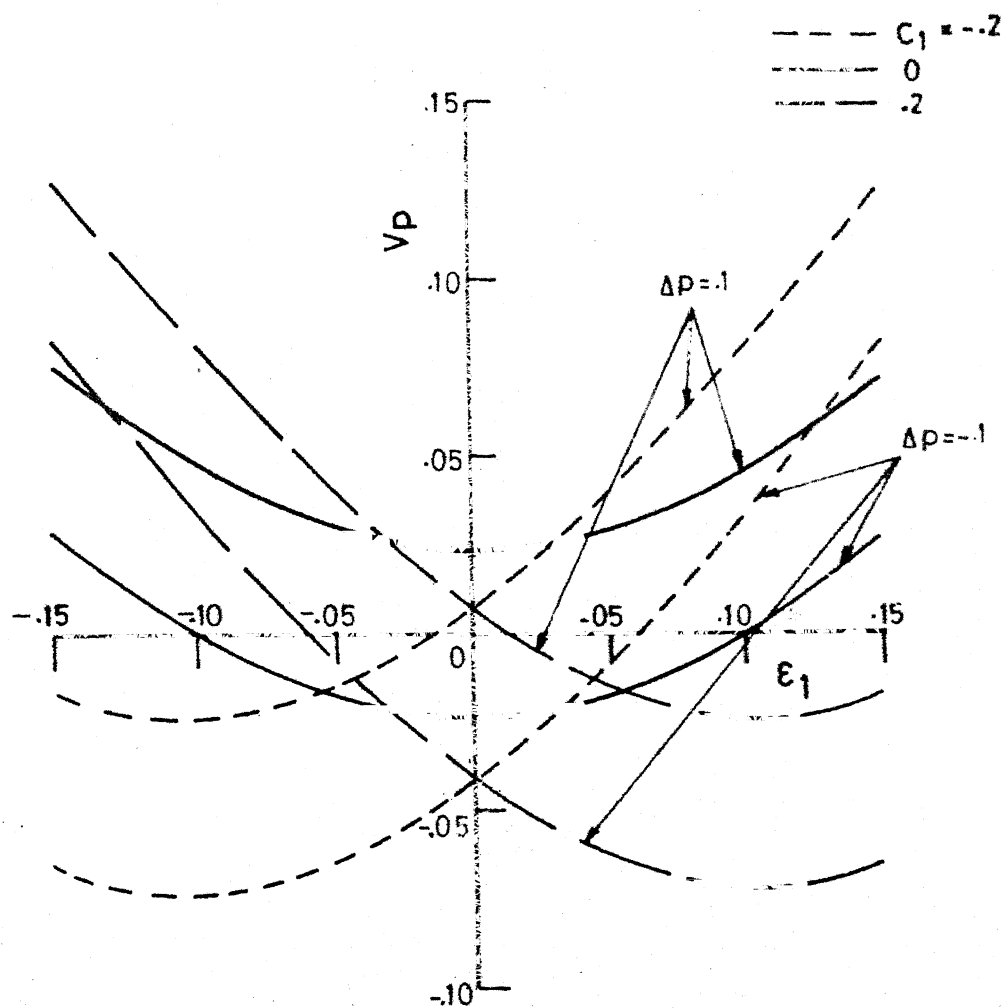
FIG. 3.16   EFFECT OF Δp AND ε ON V_p (TUBE)($\delta = 0.2$, $C_1 = 0$)

FIG. 3.17 EFFECT OF Δp AND C_1 ON Vp (TUBE)($\delta = 0.2, \xi = 0.0$)

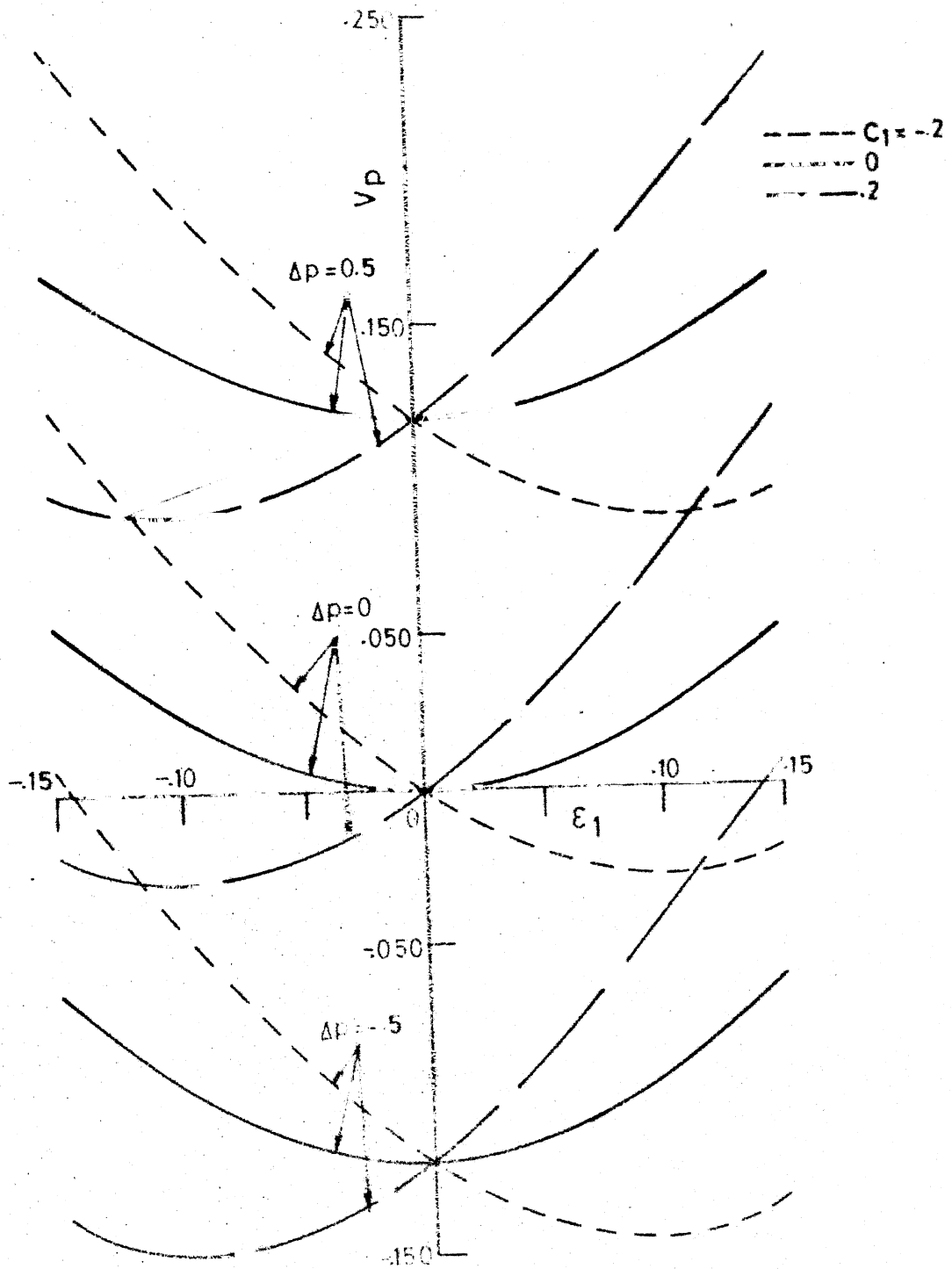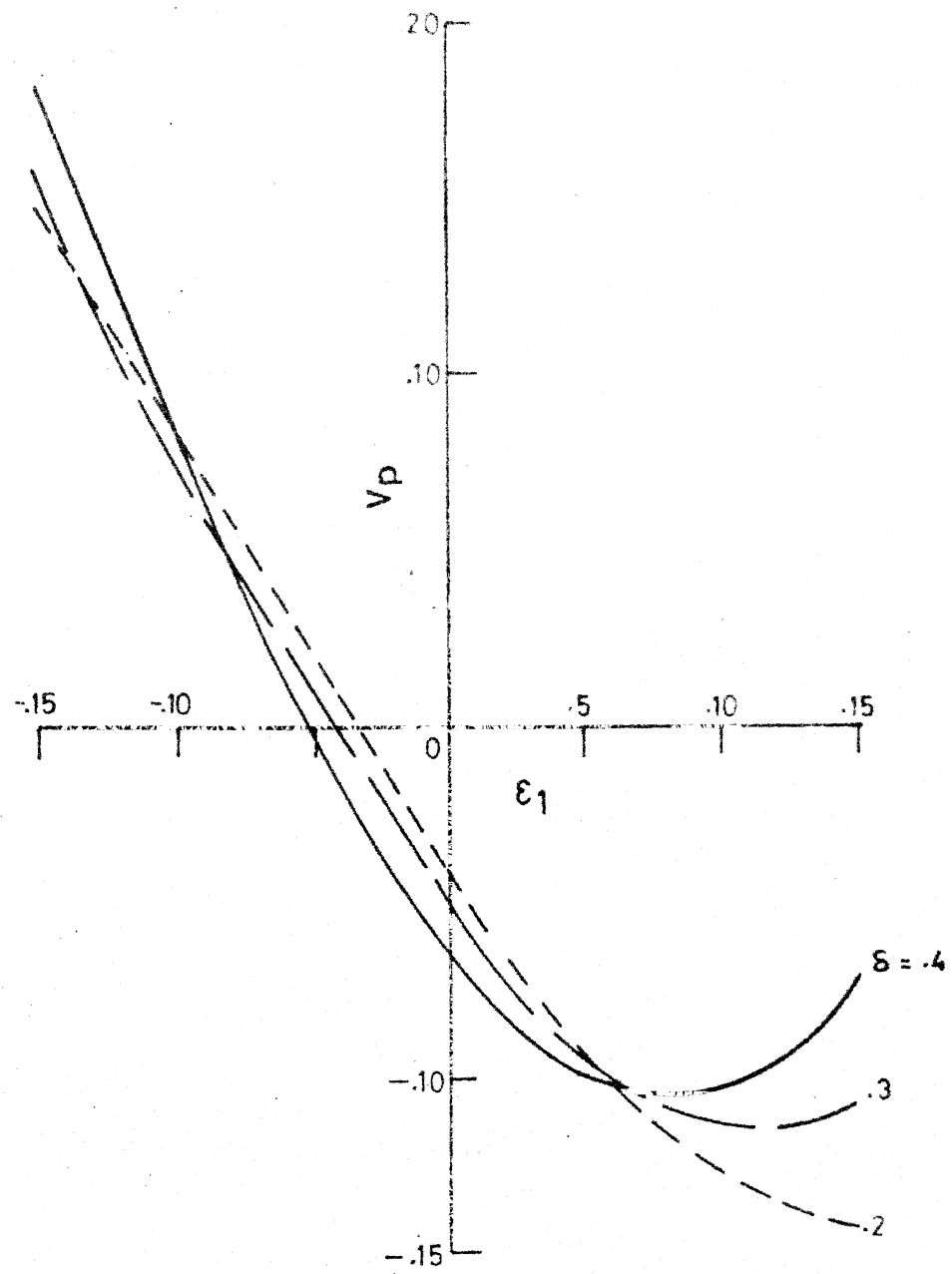FIG. 3.18   EFFECT OF δ ON V_p (TUBE) ($\Delta p = 0$, $C_1 = -2$, $\mathcal{E} = .05$)
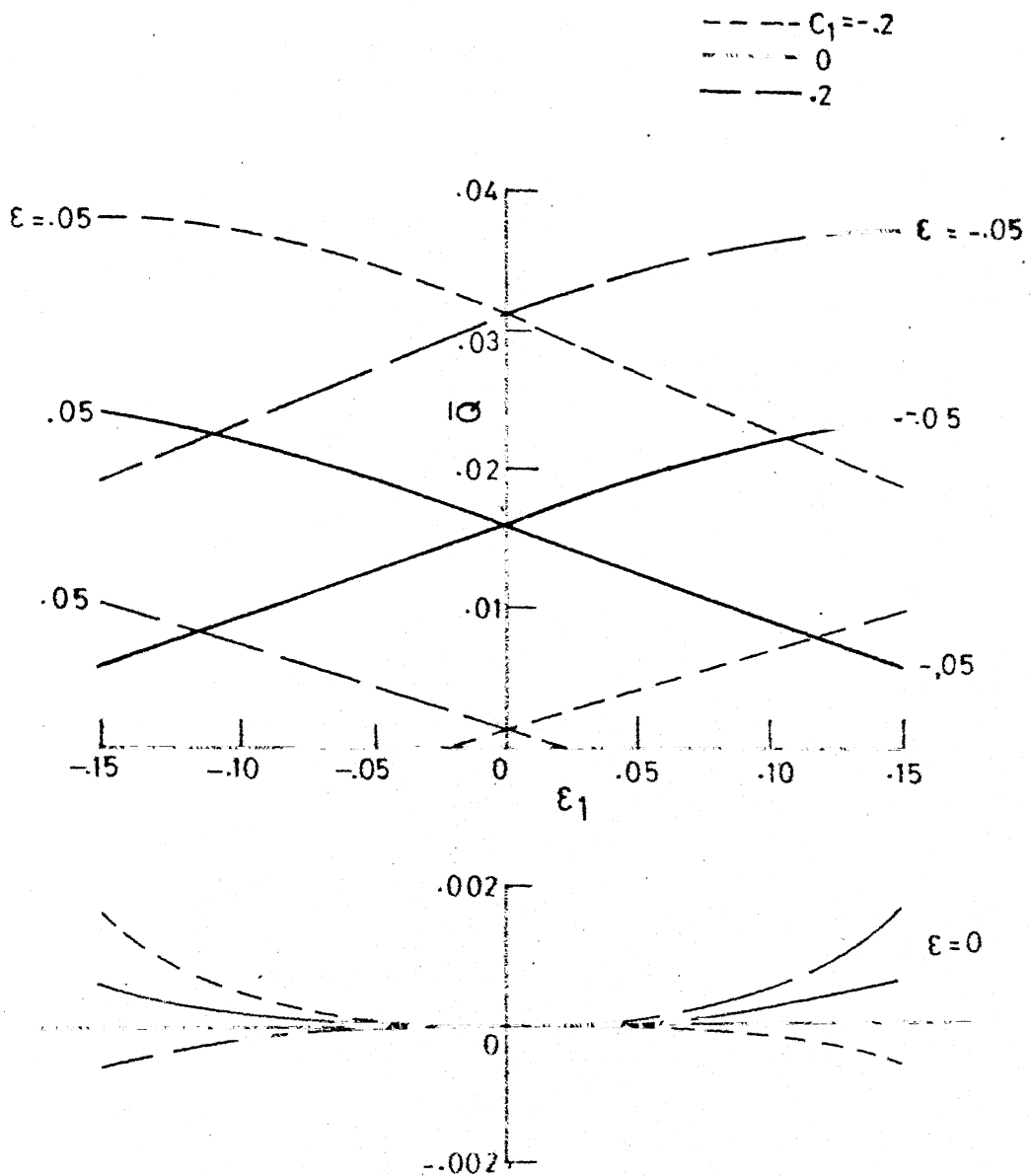
FIG 3·19 EFFECT OF C_1 AND ε ON $\bar{Q}$ (TUBE)($\delta = 0.2$, $\Delta p = 0$)
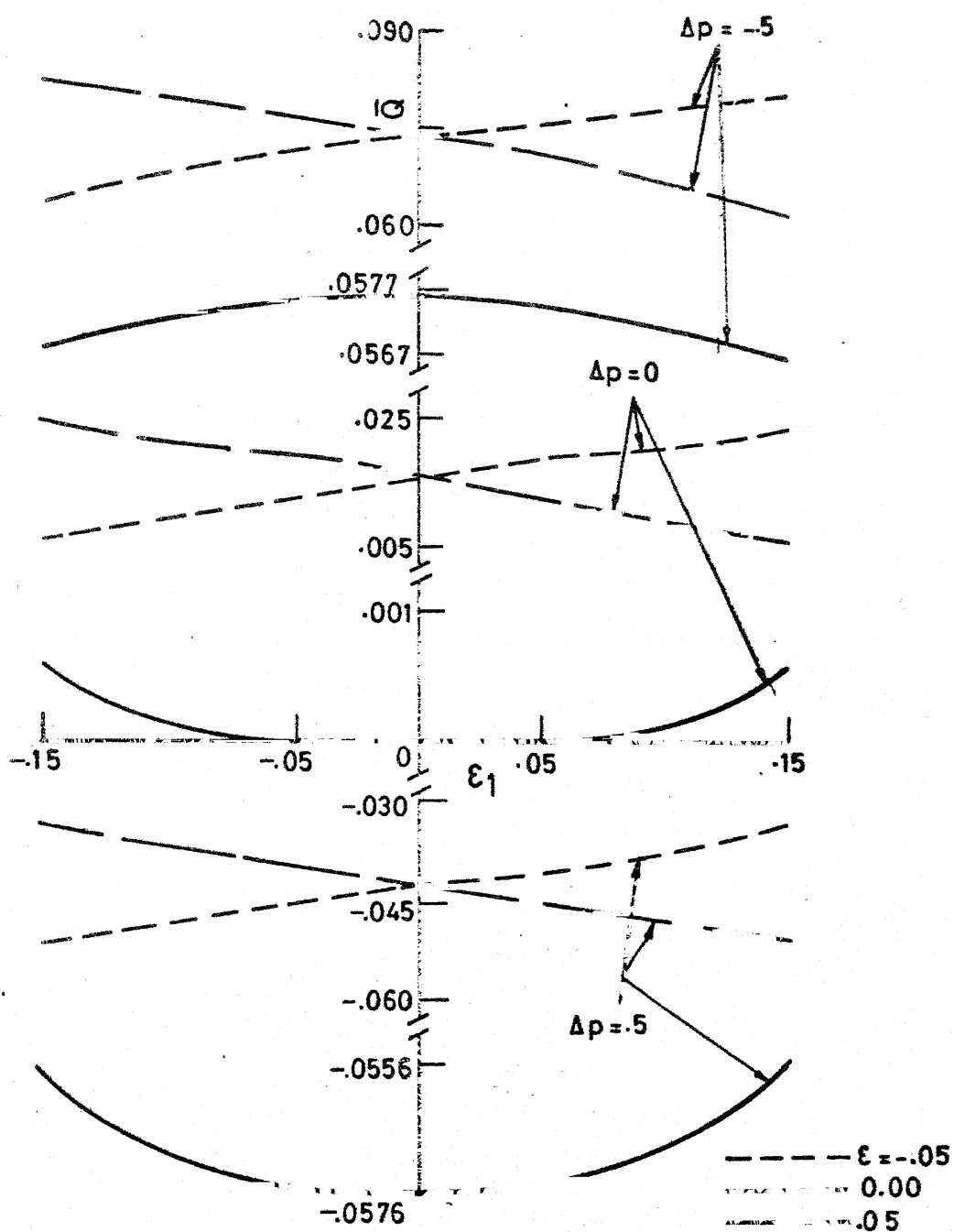
FIG. 3.20   EFFECT OF Δp AND ε ON $\bar{Q}$ (TUBE) ($\delta = 0.2, C_1 = 0$)
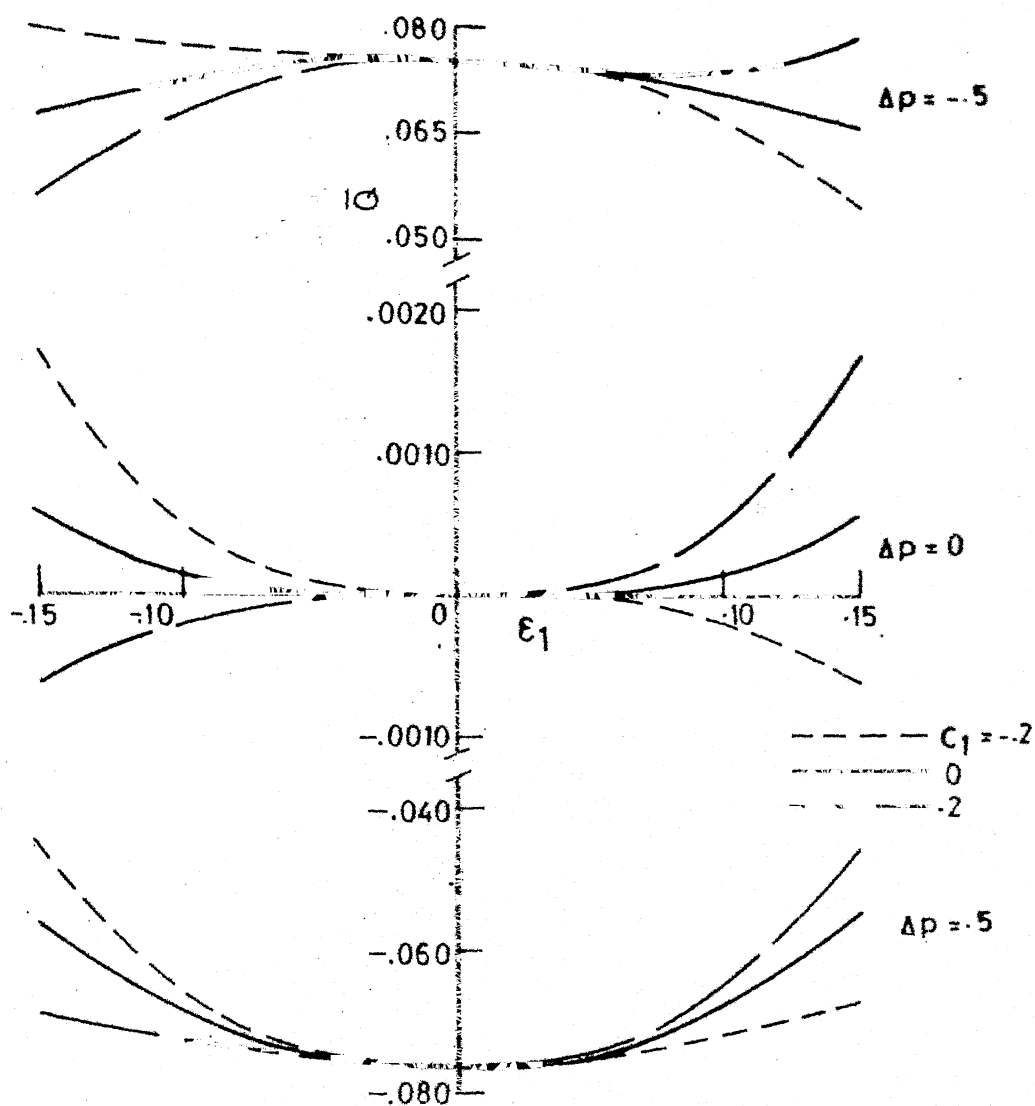
FIG. 3.21 EFFECT OF Δp AND C_1 ON $\bar{Q}$ (TUBE)($\delta = 0.2, \epsilon = 0$)
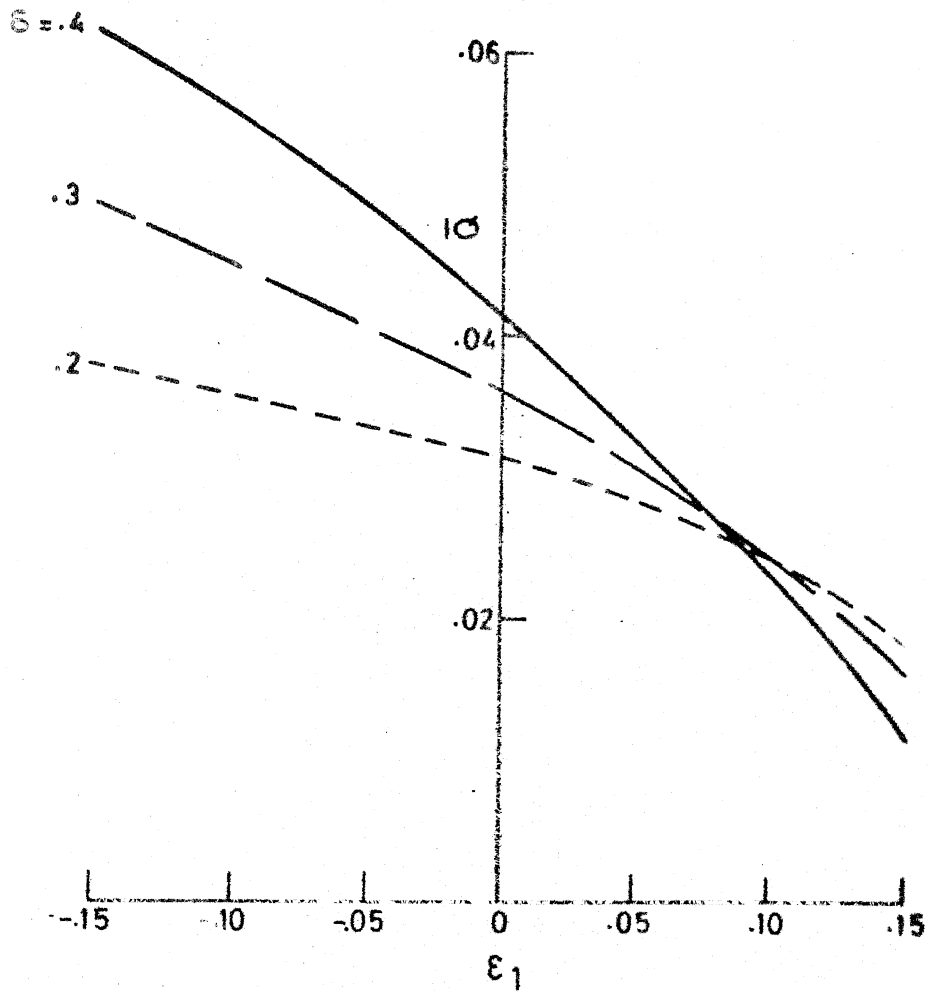
FIG. 3.22   EFEECT OF δ ON $\bar{Q}$(TUBE)$(\Delta p = 0, C_1 = -2, \mathcal{E} = .05)$

Similarly the effect of various parameters on $\bar{Q}$ can be seen from fig (3.19.)-(3.22) which is similar to those discussed in case of flow through channel.

3.4 APPLICATION TO OVUM-TRANSPORT : FERTILITY CONTROL

The ovum is the female germinal cell and is one of the largest cells in human body (diameter of ovum : 150-200 μm). At the time of ovulation the ovum is extruded from the ovary into the ovarian end of the fallopian tube and is immediately sucked in by the transverse motion of smooth muscles of the finger-like projections existing there and by the cilia beating in waves. As the life span of the unfertilized ovum is very short (approx. 18 hours) it must travel fast enough to reach the other end of the oviduct to get fertilized. It is thus evident that ovum transport along the oviduct is a vital link in the process of reproduction. In recent years there has been enough interest in the study of biochemical and physiological aspects of this problem. A review of such investigations has been presented by Pauerstein and Eddy (1979). As suggested by experimental data, cilia are necessary for ovum transport and there appears to be a correlation between the number of ciliated cells on the fimbria and fertility in humans [Brosens and Vasquez (1976), Pauerstein and Eddy (1979)]. The muscular activity in the isthmus of the oviduct is also observed to be related to ovum transport [Hodgoson et al.(1977)].
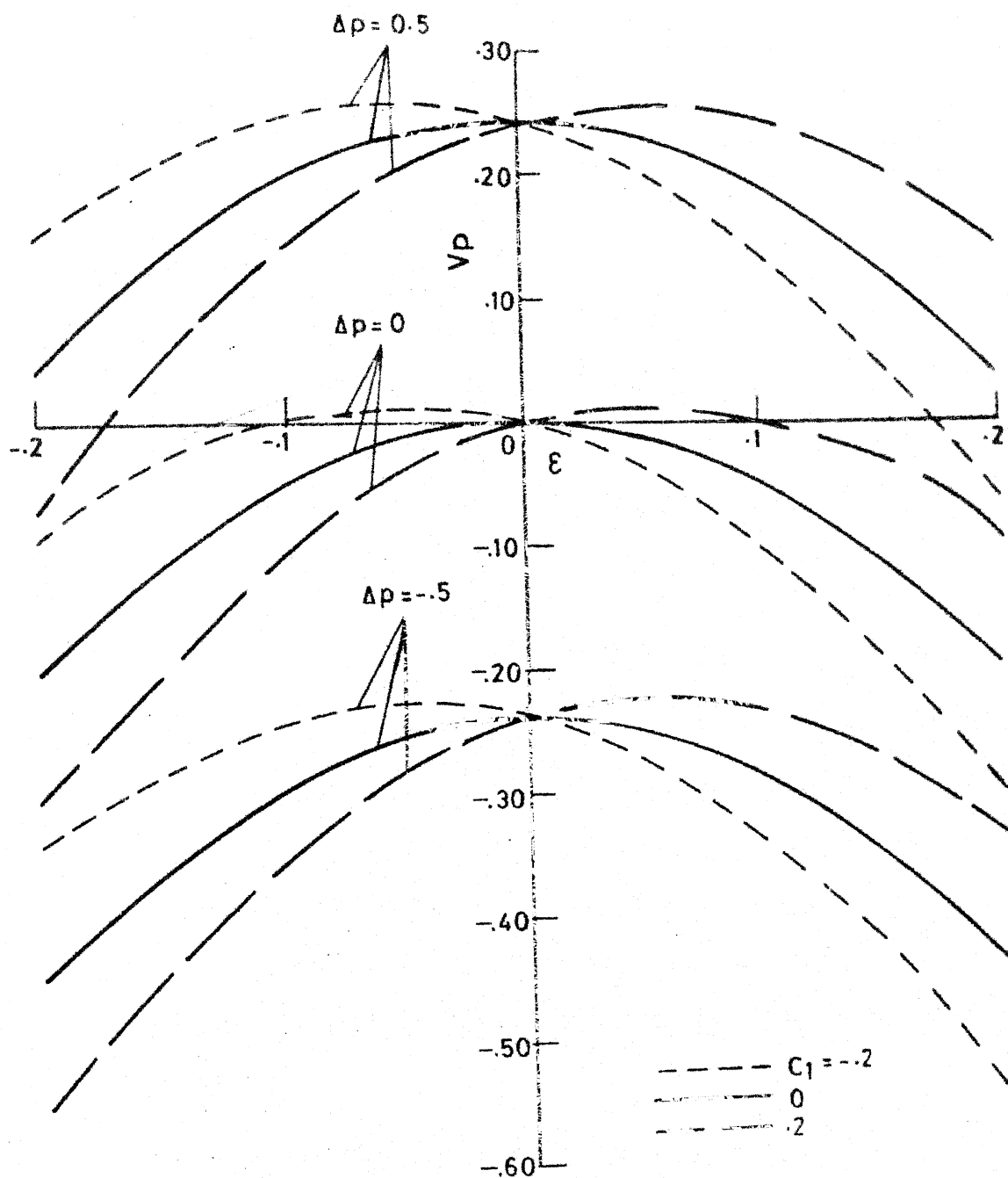
FIG. 3.23 EFFECT OF C_1 AND Δp ON VARIATION OF V_p WITH ε
($\delta = 0.2$) (OVUM TRANSPORT THROUGH A TUBE)

Recently Blake et al. (1983) studied the effect of muscular and ciliary activity along the wall of the oviduct by adding a force distribution term in the equation of motion. However, these motions can also be accounted for by assuming a combination of transverse and longitudinal motion of the walls as discussed in the above models. Since there is no evidence of ovum having its own motility, a model of the ovum transport through oviduct, accounting for the muscular and ciliary activity of the wall and the pressure difference across the oviduct, can be obtained by setting $\varepsilon_1 = 0$ in the model discussed in section 3.3.

In this case the propulsion velocity V_p has been computed using expressions (3.56) and (3.58) by putting $\varepsilon_1 = 0$. These values of V_p are plotted as function of ε in fig. 3.23 and the effects of Δp and c_1 are graphically shown. It is observed that V_p becomes more negative as the magnitude of ε is increased (i.e. the ovum is carried over by the peristaltic waves). It is also noted that V_p increases as Δp and c_1 decrease.

Since the ovum is carried along by the peristaltic waves, it is observed that its propulsion speed can be reduced if the amplitude of these waves are either minimised or are applied in the direction away from the uterus. It can therefore be suggested that for fertility control the ciliary and muscular

activity should be as minimum as possible which could be
induced by using some biochemical means such as harmones.

3.4 CONCLUSION :

Models for swimming of a micro - organism through a
channel and a tube have been proposed , taking into account
the effects of peristaltic motion of the wall and the cilia
motion. This model has been applied to ovum transport
and it has been shown that the velocity of the ovum towards
the uterus can be reduced by a reduction in the amplitude
of peristaltic and longitudinal waves. This reduction in the
propagation speed of ovum is important from the point of
fertility control, due to limited life span of unfertilized
ovum. Thus if the contraction and relaxation of smooth
muscles of the fallopian tube, causing peristaltic waves
and ciliary motion can be controled by suitable combination
of harmones, the chances of fertilization can be considerably
reduced. This aspect should therefore be explored further,
both theoretically and experimentally.

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CHAPTER IV

EFFECTS OF VISCOSITY VARIATION OF A BIOFLUID ON THE
MICRO-ORGANISMS UNDER PERISTALSIS

5.1 INTRODUCTION

/The study of self propelling micro-organisms was
initiated by Taylor (1951) and has since than attracted attention
of many researchers [Hancock (1953), Shack and Lardner (1974),
Pironneau (1975), Shukla et al. (1978)]. In chapters II and III
of this thesis this problem has been studied, taking into
account the peristaltic and longitudinal motion of the walls
of the transporting ducts which may arise due to muscular and
ciliary activities therein. These models have also been applied
to study the motion of spermatozoa and ovum. The effect of
finite thickness of the micro-organism on its motion has also
been studied in chapter III.

In these studies the effect of viscosity variation of
the fluid in which the micro-organisms swim has not been
taken into account. However it has been suggested in the
litrature that the peristaltic transport may be involved in
sperm transport through vas deferens [Guha et al. (1975), Gupta
and Seshadri (1976) Srivastava et al. (1983)] and in the
cervical canal [Smelser et al. (1974)] - all being the cases
where the viscosity of the fluid near the wall is different
from its viscosity in the center of the duct. In view of this,
a model has been presented by Shukla et al. (1980) to study

the effects of viscosity variation of the fluid on the
mechanism of peristaltic transport through a pipe and a
channel. It may be noted here that in these studies the
interaction of peristaltic waves generated by the micro-organism
on its body, to swim across the duct, with the viscosity variation
of the bio-fluid has not been studied. It is known that the
cervical mucus resembles like a close mesh during the luteal
phase and is difficult to be penetrated by the Spermatozoa
[Odeblad (1959), (1962), (1968)]. However, during the mid-period
of the menstrual cycle the micelles are aligned near the wall
[Shukla et al. (1978)] and a peripheral layer having different
viscosity is formed near the wall of the cervical canal.
Investigations has been made to utilise this behaviour of
mucus as a means of contraception [James and Marriott (1983)].

In view of the above, mathematical models have been
presented, in this chapter to study the transport of a
micro-organism in circular and rectangular ducts. under the
effect of peristaltic and longitudinal wave motion of the
walls and the viscosity variation of the bio-fluid along
the cross-section. The equations of motions and the force
equilibrium conditions applied in chapter III are modified
to take into account the visvosity variation. As in
chapter III the solutions have been obtained for this general
case. A particular case when viscosity is a step function
has been discussed. It has been shown that, apart from

the results obtained in chapter III, the propulsive velocity
of the micro-organism decreases as the viscosity of the
peripheral layer increases.

4.2 TRANSPORT THROUGH A CHANNEL

4.2.1 MATHEMATICAL MODEL

Consider the propulsion of a flexible slab
(micro-organism) of infinite width and finite thickness
$2\delta'$ in the middle of a channel of thickness 2a. The space
between boundaries of the slab and the channel is filled
with a Newtonian fluid with varying viscosity. The walls
of the channel are undergoing both peristaltic and longitudinal
wave motion. It is further assumed that the slab is being
propelled at a constant speed in the negative X'-direction,
in a stationary frame (X', Y'), by propagating peristaltic
waves along it surfaces. If these waves travel with the
speed $c-V'_p$ in the stationary frame then in this frame the
shapes of the walls of the channel $(Y' = \pm H')$ and the
slab $(Y' = \pm H'_1)$ at any instant t' are given by (see fig. 4.1)

$$H'(X', t') = a + b' \sin \frac{2\pi}{\lambda} (X' - ct' + V'_p t') \qquad (4.1)$$

$$H'_1(X', t') = \delta' + b'_1 \sin \frac{2\pi}{\lambda}(X' - ct' + V'_p t') \qquad (4.2)$$

where b' and b'_1 are the amplitudes of waves on boundaries
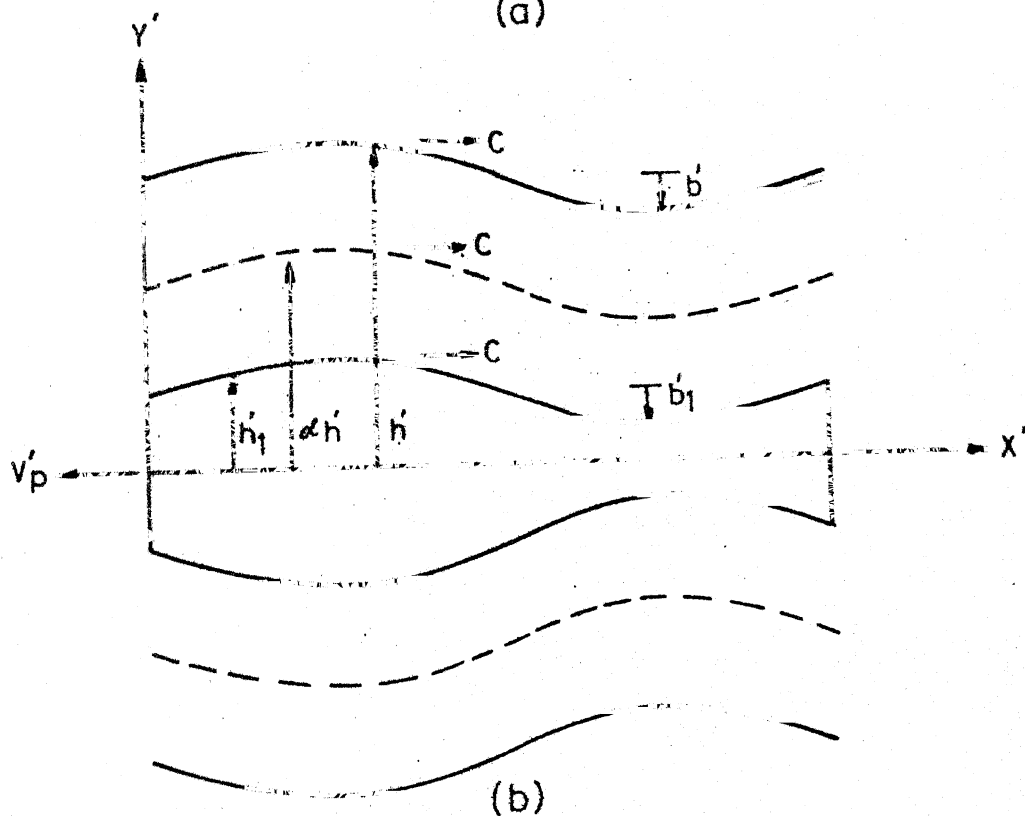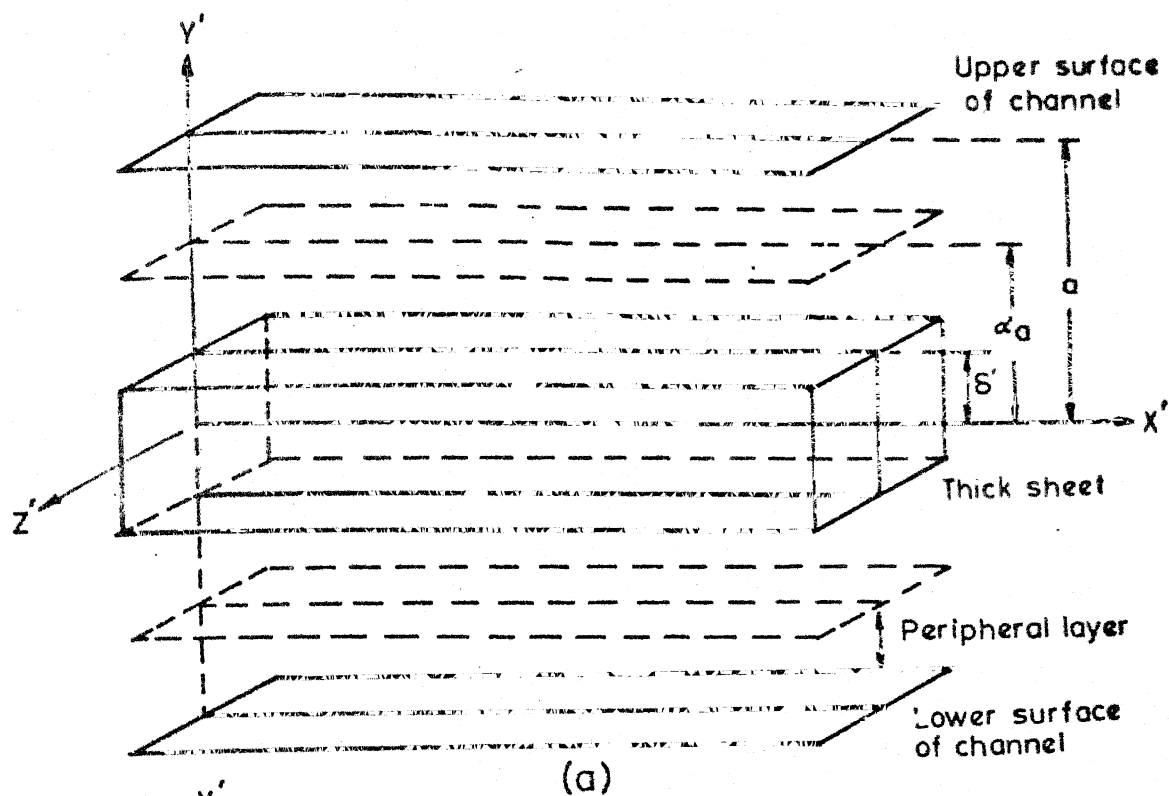of the channel and the slab respectively.

FIG. 4.1 FLOW OF FLUID IN CHANNEL WITH PERIPHERAL LAYER

(a) Three dimensional without peristalsis

(b) Two dimensional with peristalsis

The equations governing the flow of the fluid through the gap $(H_1' \leq Y' \leq H')$, under long wavelength approximation and for low Reynolds number conditions, can be written as

$$0 = - \frac{\partial P'}{\partial X'} + \frac{\partial}{\partial Y'}(\mu' \frac{\partial U'}{\partial Y'}) \tag{4.3}$$

$$0 = - \frac{\partial P'}{\partial Y'} \tag{4.4}$$

$$\frac{\partial U'}{\partial X'} + \frac{\partial V'}{\partial Y'} = 0 \tag{4.5}$$

where U' and V' are the velocity components in X' and Y' directions respectively, P' is the pressure and $\mu' = \mu'(Y')$ is the viscosity of the fluid which varies along the cross-section of the channel.

The boundary conditions are same as in section 3.2, i.e.

$$U' = G'(X' - ct' + V_p' t') = c_1' \sin \frac{2\pi}{\lambda}(X' - ct' + V_p' t)$$

$$\text{at } Y' = \pm H'(X', t') \tag{4.6}$$

$$U' = -V_p' \quad \text{at } Y' = \pm H_1'(X', t')$$

Force equilibrium condition on the surface of the micro-organism is

$$\int_S \tau \, dS = \delta' \, \Delta P' \tag{4.7}$$

where τ is the resultant of the force acting on the

surface S, of the micro-organism and Δp is the pressure rise over a wavelength.

Applying following transformation

$$x = (X'-ct'+V_p' t')/\lambda, \quad y = Y'/a, \quad t = ct'/\lambda$$

$$u = (U'-c+V_p')/c, \quad v = \lambda V'/ac, \quad (g,c_1,V_p) = (G',c_1',V_p')/c$$

$$p = p' a^2/\lambda c \mu_1, \quad \mu = \mu'/\mu_o, \quad (\varepsilon,\varepsilon_1,\delta) = (b',b_1',\delta')/a \qquad (4.8)$$

where $\mu_o = \mu'(0)$, the eqn. of motion (4.3)-(4.5) can be written in dimensionless form of wave frame moving with a velocity $c-V_p'$, as

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \qquad (4.9)$$

$$0 = -\frac{\partial p}{\partial Y} \qquad (4.10)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (4.11)$$

The boundary conditions (4.6) are transformed as

$$u = g(x) + V_p - 1 \quad \text{at } y = h(x)$$
$$ = -1 \qquad\qquad \text{at } y = h_1(x) \qquad (4.12)$$

where

$$h(x) = 1 + \varepsilon \sin 2\pi x$$

$$h_1(x) = \delta + \varepsilon_1 \sin 2\pi x \qquad (4.13)$$

$$g(x) = c_1 \sin 2\pi x$$

The force equilibrium condition, in this case becomes

$$\int_0^1 \left[(\mu \frac{\partial u}{\partial y})_{y=h_1(x)} + \frac{dh_1(x)}{dx} p\right]dx = \delta \Delta p \qquad (4.14)$$

4.2.2 ANALYSIS

The equation (4.9) can be solved, using boundary conditions (4.12), as

$$u = -1 + \left(\frac{\partial p}{\partial x}\right)\left[I_2(y) - \frac{I_2(h)}{I_1(h)}I_1(y)\right] + (g+V_p)\frac{I_1(y)}{I_1(h)} \qquad (4.15)$$

where

$$I_1(y) = \int_{h_1}^{y} \frac{dy}{\bar{\mu}(y)}$$

$$I_2(y) = \int_{h_1}^{h} \frac{y}{\bar{\mu}(y)} \, dy \qquad (4.16)$$

$$I_3(y) = \int_{h_1}^{y} \frac{y^2}{\bar{\mu}(y)} \, dy$$

The flux in the moving frame is obtained as

$$q = \int_{h_1}^{h} u \, dy = -(h-h_1) - \left(\frac{\partial p}{\partial x}\right)\left[I_3(h) - \frac{I_2^2(h)}{I_1(h)}\right] + (g+V_p)\left[h - \frac{I_2(h)}{I_1(h)}\right]$$

$$(4.17)$$

which gives $-\frac{\partial p}{\partial x}$ as

$$-\left(\frac{\partial p}{\partial x}\right) = \frac{1}{I_3(h) \cdot I_1(h) - I_2^2(h)}\left[\{q+(h-h_1)\}I_1(h) + (g+V_p)\{I_2(h) - hI_1(h)\}\right]$$

$$(4.18)$$

Integrating (4.18) we get

$$-\Delta p = F_{11}q + F_{12}V_p + F_{13}c_1 + F_{14} \qquad (4.19)$$

where

$$F_{11} = \int_{o}^{1}\left[-\frac{I_1(h)}{I_1(h)I_3(h) - I_2^2(h)}\right]dx$$

$$F_{12} = \int_0^1 \left[\frac{I_2(h) - hI_1(h)}{I_1(h)I_3(h) - I_2^2(h)} \right] dx \qquad (4.20)$$

$$F_{13} = \int_0^1 \left[\frac{I_2(h) - hI(h)}{I_1(h)I_3(h) - I_2^2(h)} \right] \sin 2\pi x \, dx$$

$$F_{14} = \int_0^1 \left[\frac{(h-h_1)I_1(h)}{I_1(h)I_3(h) - I_2^2(h)} \right] dx$$

Using expressions (4.15) and (4.17) for u and p, the force equilibrium condition becomes

$$O = F_{21}q + F_{22}V_p + F_{23}c_1 + F_{24} \qquad (4.21)$$

where

$$F_{21} = \int_0^1 \left[\frac{I_2(h)}{I_1(h)I_3(h) - I_2^2(h)} \right] dx$$

$$F_{22} = \int_0^1 \left[\frac{I_3(h) - hI_2(h)}{I_1(h)I_3(h) - I_2^2(h)} \right] dx \qquad (4.22)$$

$$F_{23} = \int_0^1 \left[\frac{I_3(h) - hI_2(h)}{I_1(h)I_3(h) - I_2^2(h)} \right] \sin 2\pi x \, dx$$

$$F_{24} = \int_0^1 \left[\frac{(h-h_1)I_2(h)}{I_1(h)I_3(h) - I_2^2(h)} \right] dx$$

The equations (4.19) and (4.21) are linear in q and V_p and can be solved, therefore, as

$$V_p = \frac{M_{11}\Delta p + M_{12}c_1 + M_{13}}{M_{oo}}$$

$$q = -\frac{M_{21}\Delta p + M_{22}c_1 + M_{23}}{M_{oo}} \qquad (4.23)$$

where

$$M_{oo} = F_{22}F_{11} - F_{12}F_{21}$$

$$M_{11} = F_{21}$$

$$M_{12} = F_{13}F_{21} - F_{23}F_{11}$$

$$M_{13} = F_{14}F_{21} - F_{24}F_{11} \qquad (4.24)$$

$$M_{21} = F_{22}$$

$$M_{22} = F_{13}F_{22} - F_{23}F_{12}$$

$$M_{23} = F_{14}F_{22} - F_{24}F_{12}$$

The time averaged flux $\bar{Q}$ in stationary can be obtained from the following relation

$$\bar{Q} = q + (1-V_p)(1-\alpha) \qquad (4.25)$$

Eqns. (4.23)-(4.25) determing the propulsion velocity V_p and the time averaged flux in the fixed coordinate system for any viscosity function $\mu(y)$. In the following section effect of a peripheral layer is considered as a particular case when $\mu(y)$ is a step function.

4.2.3 EFFECT OF PERIPHERAL LAYER

To study the effect of peripheral layer the viscosity function has been assumed in following form

$$\mu(y) = \begin{array}{l} = \bar{\mu} \quad ; \quad \alpha h \leq y \leq h \\ = 1 \quad ; \quad h_1 \leq y \leq \alpha h \end{array} \qquad (4.26)$$

where $(1-\alpha)h$ is the thickness of the peripheral layer near the wall of the channel.

With this viscosity function, the integrals
$I_1(h)$, $I_2(h)$, $I_3(h)$ can be analytically evaluated as follows

$$I_1(h) = \frac{1}{\bar{\mu}}(h-\alpha h) + (\alpha h - h_1)$$

$$I_2(h) = \frac{1}{2\bar{\mu}}(h^2 - \alpha^2 h^2) + \frac{1}{2}(\alpha^2 h^2 - h_1^2) \qquad (4.27)$$

$$I_3(h) = \frac{1}{3\bar{\mu}}(h^3 - \alpha^3 h^3) + \frac{1}{2}(\alpha^3 h^3 - h_1^3)$$

It can be seen from (4.27) that this model reduces
to the model discussed in section 3.2 of chapter III, when
there is no peripheral layer i.e., $\alpha = 1$ or $\bar{\mu} = 1$.

4.2.4. RESULTS AND DISCUSSIONS

Assuming $\mu(y)$ as a step function (eqn. (4.26)), to
account for the presence of peripheral layer, V_p and $\bar{Q}$ have
been calculated from expressions (4.21), (4.23)-(4.25), (4.27).
These results are graphically shown in fig. 4.2-4.7. It
can be observed that effects of c_1 and ε on V_p (fig.4.2)
and $\bar{Q}$ (fig. 4.5) are similar to those obtained in section 3.2
of chapter III, i.e. V_p decreases and $\bar{Q}$ increases as ε and
c_1 increase. Effect of $\bar{\mu}$ and α on V_p and $\bar{Q}$ can be studied
from figs. 4.3 and 4.6 respectively. It is noted here that
magnitudes of V_p and $\bar{Q}$ decrease as α and $\bar{\mu}$ increase. Effect
of Δp is also similar as observed in chapter III i.e. as Δp
increases from negative to positive values. V_p becomes more
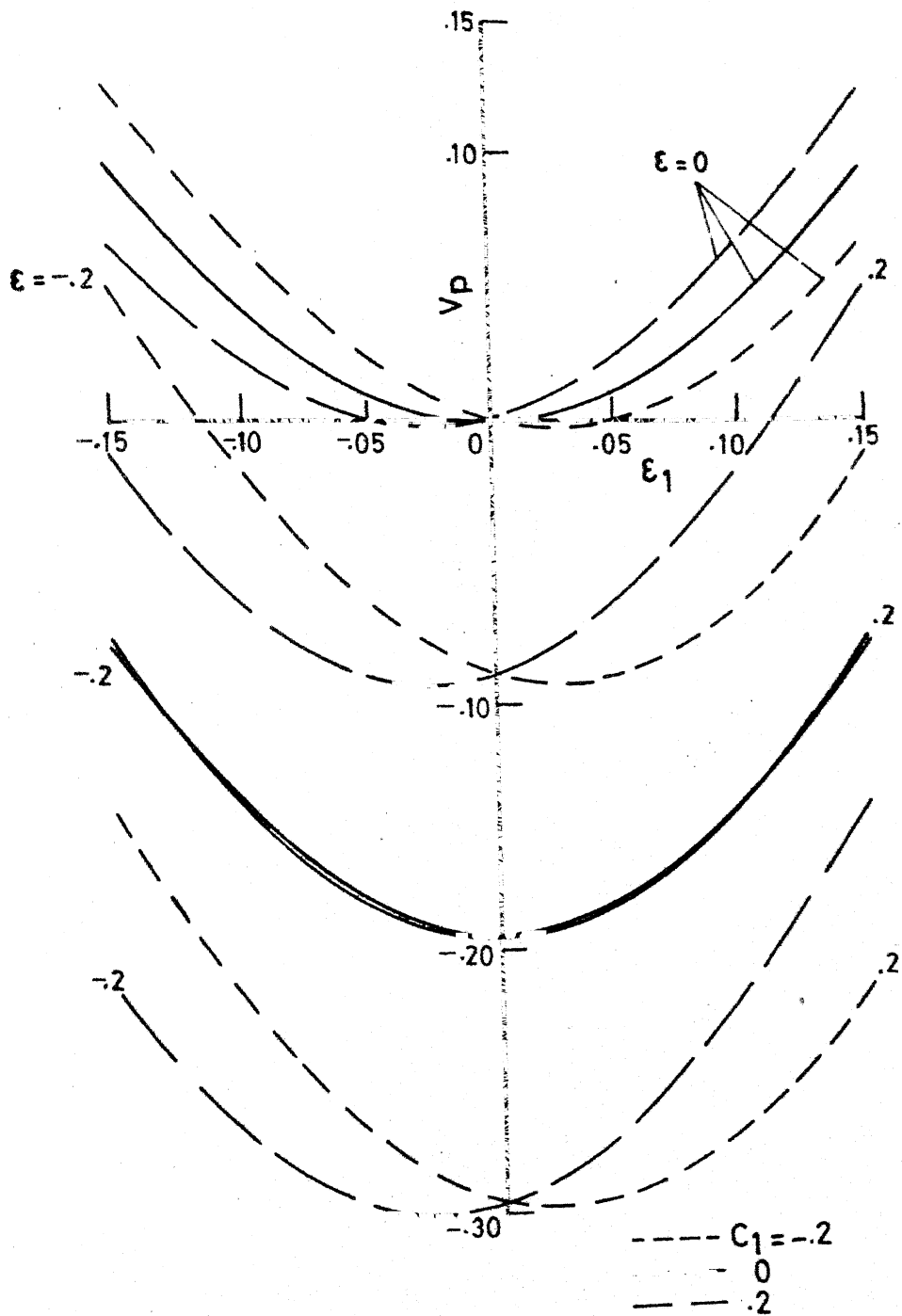negative (fig. 4.4) and $\bar{Q}$ becomes more positive (fig.4.7).

FIG. 4.2 EFFECT OF C_1 AND ε ON VARIATION OF V_P WITH ε_1 (CHANNEL) ($\delta = 0.2$, $\Delta p = 0.0$, $\bar{\mu} = 0.75$, $\alpha = 0.90$)
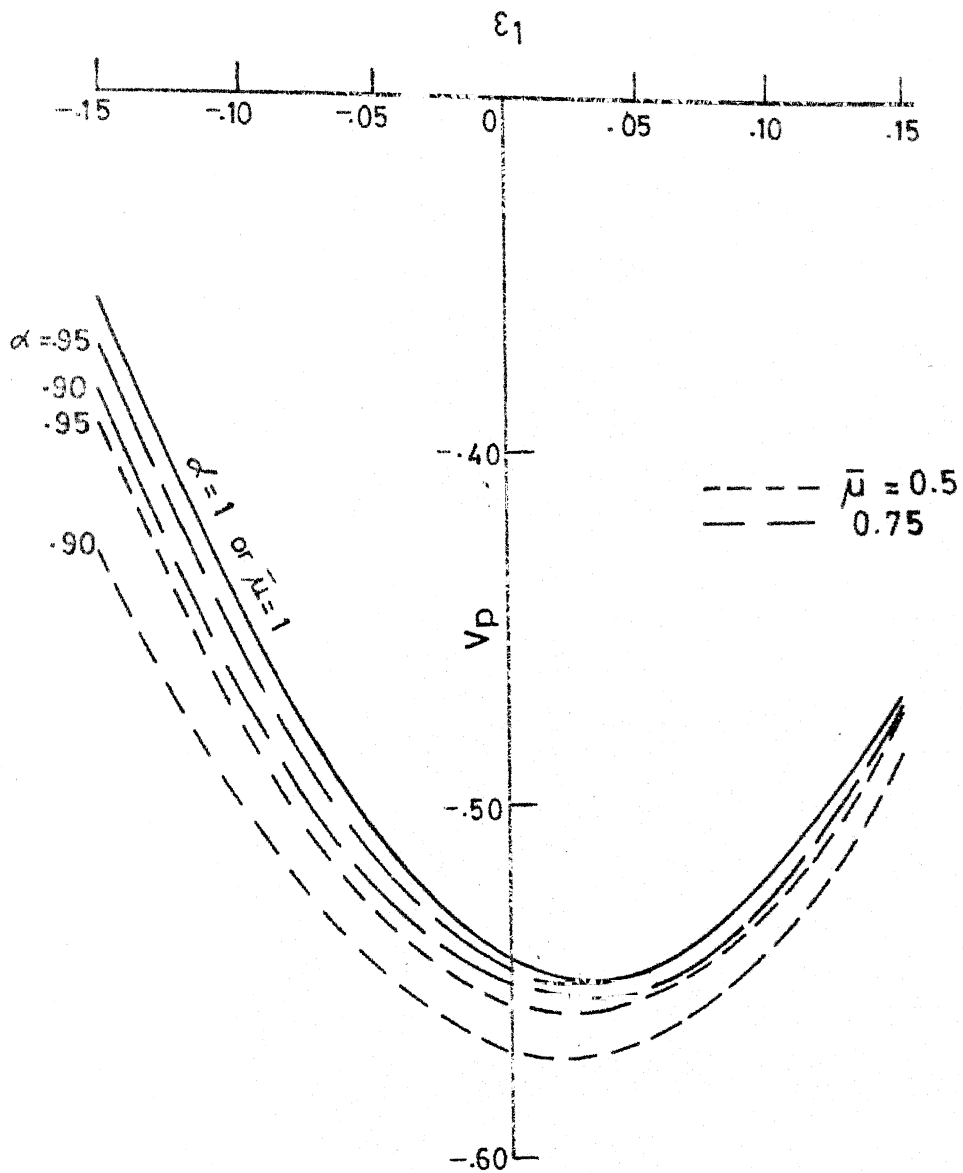
FIG. 4.3 EFFECT OF α AND $\bar{\mu}$ ON V_p (CHANNEL) ($C_1 = -.2$, $\varepsilon = .2$, $\Delta p = -.5$, $\delta = .2$)
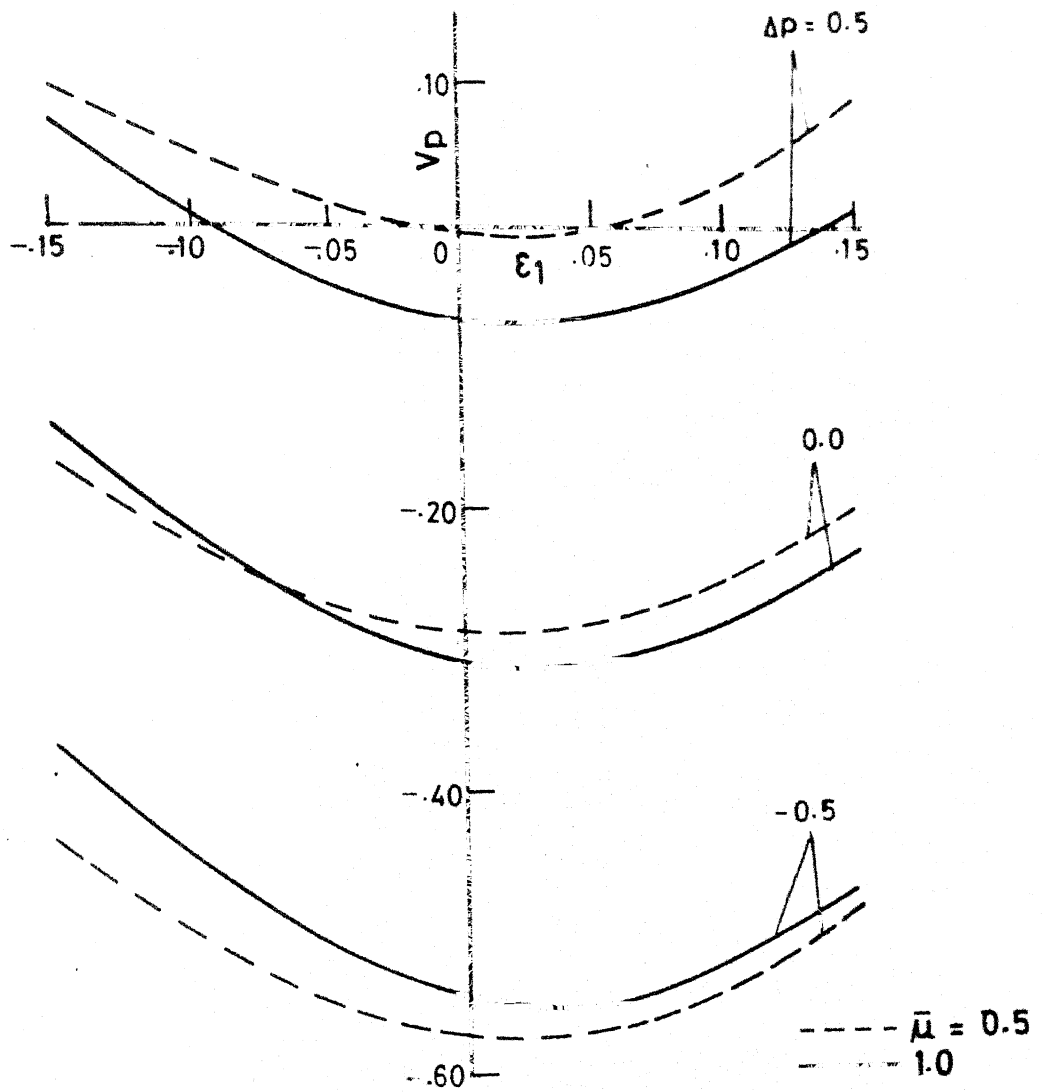
FIG. 4.4 EFFECT OF $\bar{\mu}$ AND Δp ON VARIATION OF V_p WITH ε_1
(CHANNEL) ($\delta = 0.2$, $C_1 = -.2$, $\varepsilon = -.2$, $\alpha = .90$)
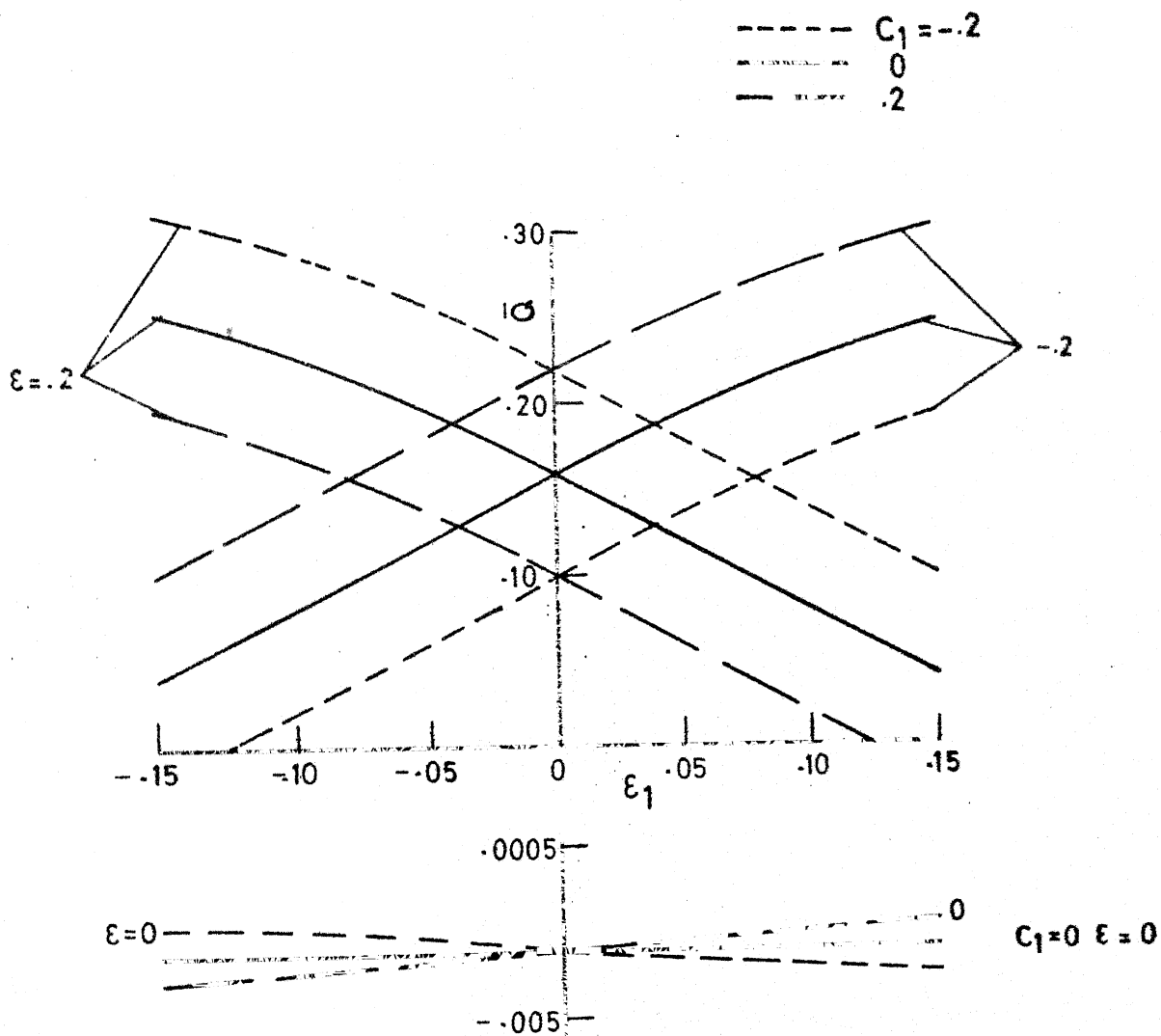
FIG.4·5   EFFECT OF C_1 AND ε ON $\bar{Q}$ (CHANNEL) ($\delta = 0.2$, $\Delta p = 0$, $\bar{\mu} = 0.75$, $\alpha = 0.90$)
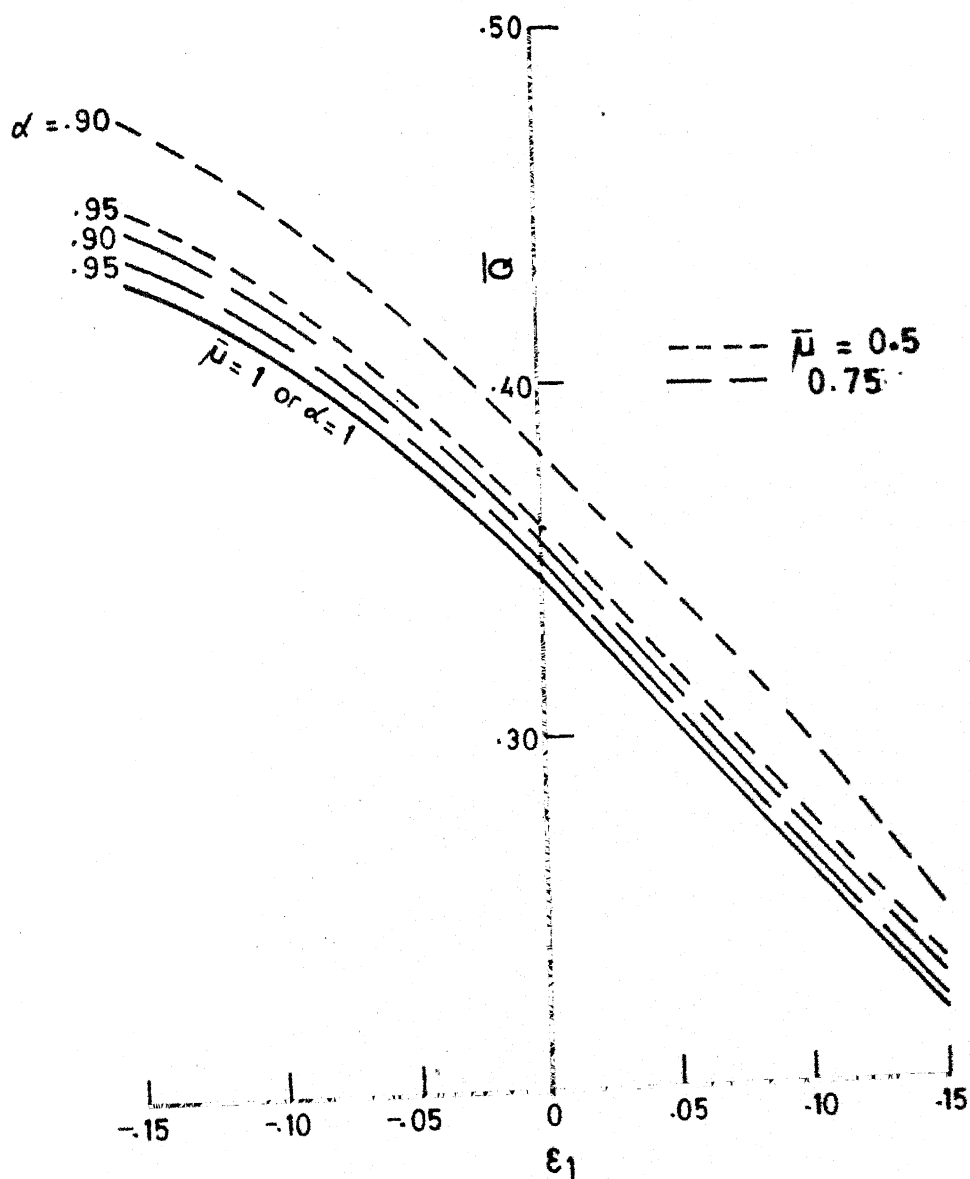
FIG 4.6 EFFECT OF α AND $\bar{\mu}$ ON $\bar{Q}$ (CHANNEL) ($C_1 = -.2$, $\varepsilon = .2$, $\Delta p = -.5$, $\delta = .2$)
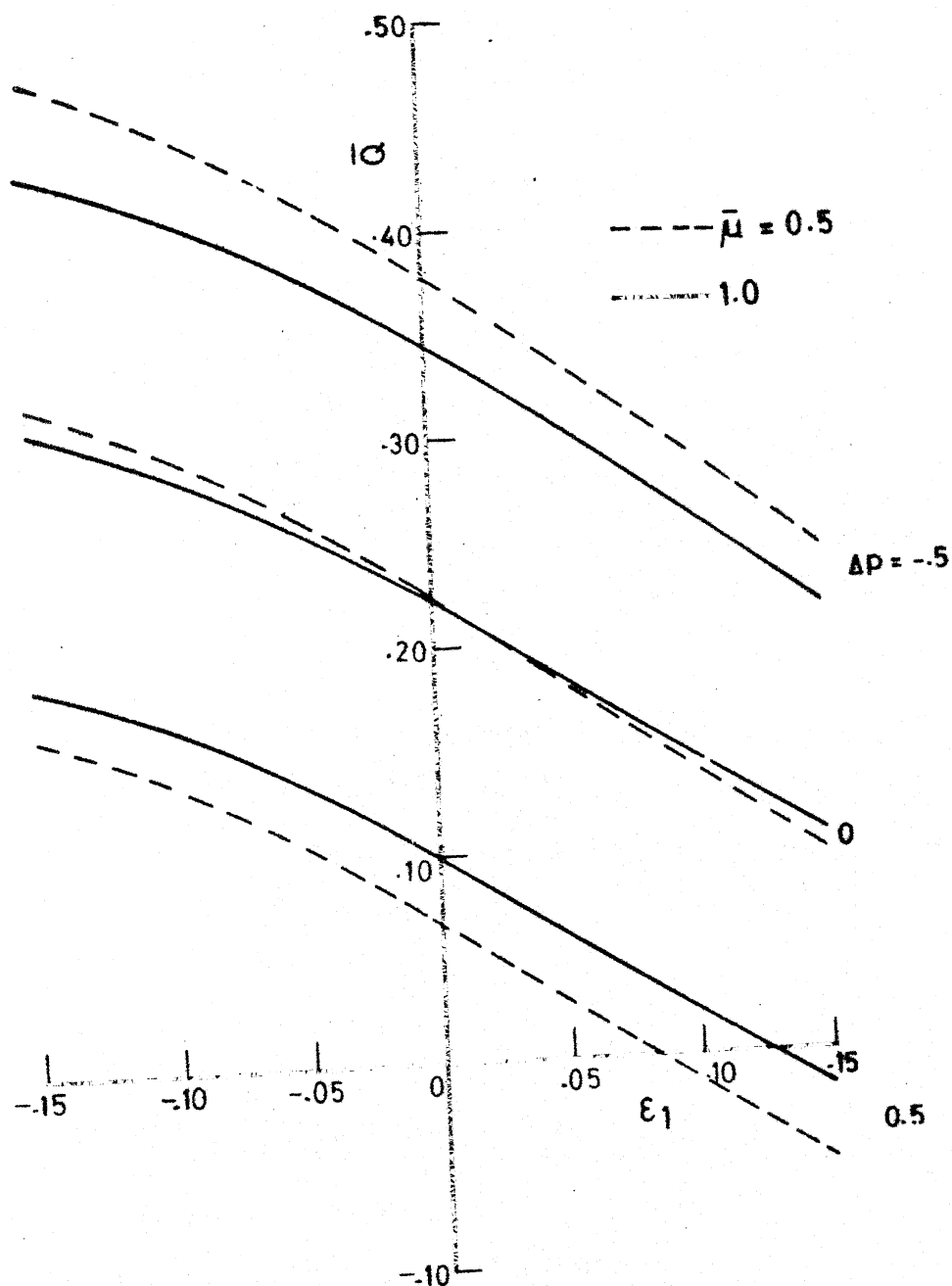
FIG. 4.7 EFFECT OF $\bar{\mu}$ AND Δp ON VARIATION OF $\bar{Q}$ WITH ε_1
(CHANNEL) ($\delta = 0.2$, $C_1 = -2$, $\varepsilon = 2$ $\alpha = 90$)

4.3 TRANSPORT THROUGH A TUBE

4.3.1 MATHEMATICAL MODEL

Consider the flow of an incompressible Newtonian fluid, whose viscosity varies along the cross-section of the tube, in a tube of radius a, where the central core region (fig.4.8) of radius δ' is moving with a velocity V'_p in the negative axial direction. The fluid flow is confined only to the region between the boundaries of the core and the tube. Let a peristaltic wave of wavelength λ and amplitude b' pass over the wall of the tube with speed $c-V'_p$. It is also assumed that a peristaltic wave of same speed and wavelength and amplitude b' passes over the boundary of the cylinderical core. Then the shapes of the boundaries of the tube H' and the core H'_1, at instant t' are given by

$$H'(X',t') = a + b'\sin\frac{2\pi}{\lambda}(X'-ct'+V'_p t') \qquad (4.28)$$

$$H'_1(X',t') = \delta' + b'_1 \sin\frac{2\pi}{\lambda}(X'-ct'+V'_p t') \qquad (4.29)$$

For low Reynolds number the equations governing the flow, under long wavelength approximation are given by

$$0 = -\frac{\partial P'}{\partial X'} + \frac{1}{R'}\frac{\partial}{\partial R'}\left(\mu R'\frac{\partial U'}{\partial R'}\right) \qquad (4.30)$$

$$0 = -\frac{\partial P'}{\partial R'} \qquad (4.31)$$

$$\frac{\partial U'}{\partial X'} + \frac{1}{R'}\frac{\partial}{\partial R'}(R'V') = 0 \qquad (4.32)$$
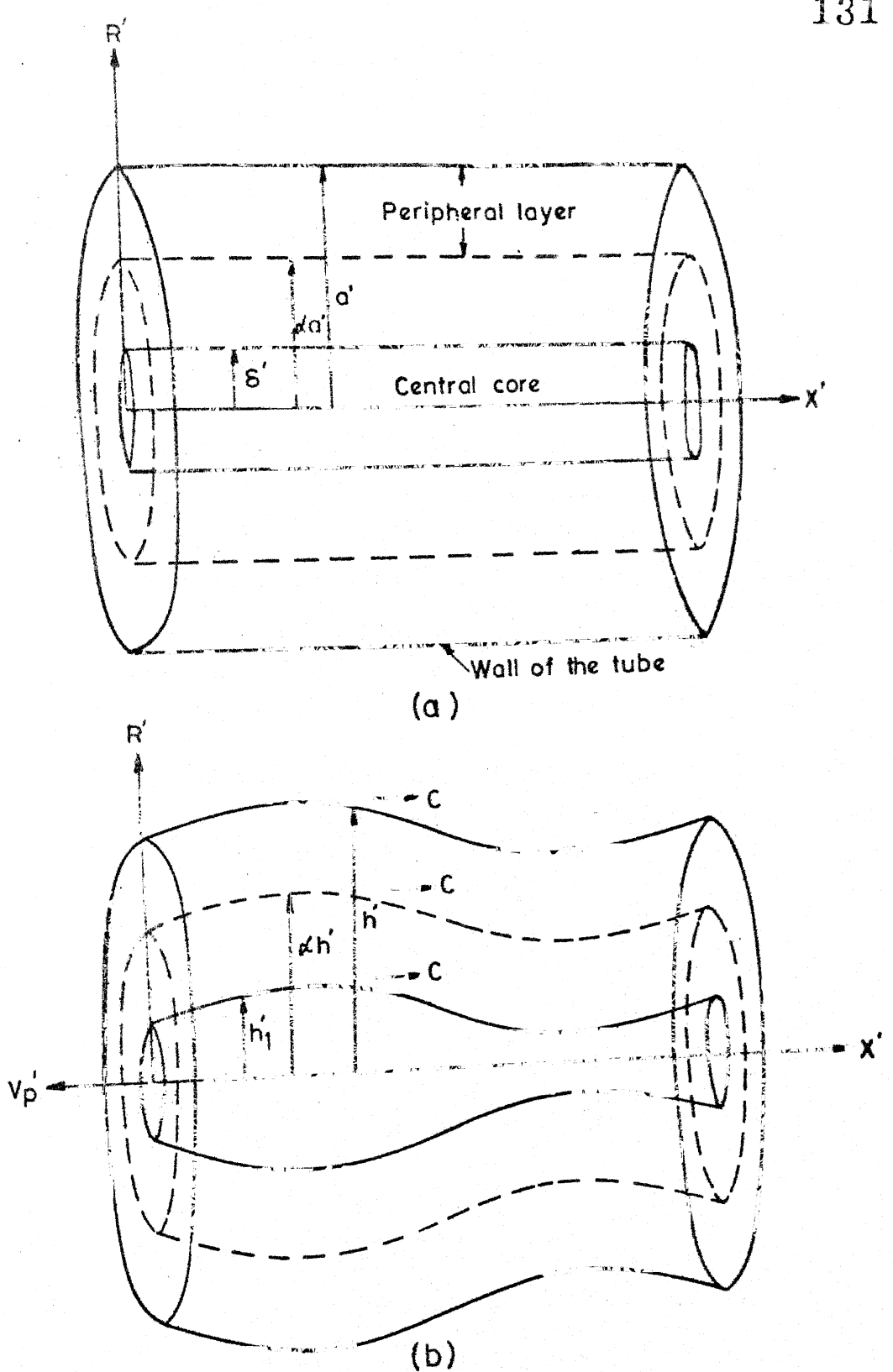
FIG. 4.8 FLOW OF FLUID IN TUBE WITH PERIPHERAL LAYER
(a) Without peristalsis   (b) With peristalsis

Considering the longitudinal motility of the wall
of the tube the boundary conditions are

$$U' = G_1'(X'-ct'+V_p't') = c_1\sin\frac{2\pi}{\lambda}(X'-ct'+V_p't')$$

$$\text{at } R' = H'(X',t') \tag{4.33}$$

$$U' = -V_p' \quad \text{at } R' = H_1'(X',t')$$

Transforming all the quantities to wave frame
(moving with velocity $c-V_p'$ in the positive axial direction)
and non-dimensionalising them with following relations

$$x = (X'-ct'+V_p't')/\lambda \ , \ r = R'/a \ , \ t = t'c/\lambda$$

$$u = (U'-c-V_p') \ , \quad (g,c_1,V_p) = (G',c_1',V_p')/c$$

$$v = V'\lambda/ca, \quad p = p'a^2/\mu_1(\lambda, \ \mu = \mu'/\mu_1$$

$$(\varepsilon,\varepsilon,\delta) = (b',b_1',\delta')/a \tag{4.31}$$

(where μ_1 is some characteristic viscosity, the equations
(4.30)-(4.32) can be written as

$$0 = -\frac{\partial p}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(\mu r\frac{\partial u}{\partial r}) \tag{4.35}$$

$$0 = -\frac{\partial p}{\partial r} \tag{4.36}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0 \tag{4.37}$$

and the boundary conditions (4.33) are transformed as

$$u = g + V_p-1 \quad ; \ r = h(x)$$
$$= -1 \quad ; \ r = h_1(x) \tag{4.38}$$

where

$$g(x) = c_1 \sin 2\pi x$$

$$h(x) = 1 + \varepsilon \sin 2\pi x \qquad (4.39)$$

$$h_1(x) = \delta + \varepsilon_1 \sin 2\pi x$$

The force equilibrium condition, in this case is

$$2\pi \int_0^1 \left[\left(\mu r \frac{\partial u}{\partial r} \right)_{r=h_1} + h_1(x) \frac{dh_1(x)}{dx} p \right] dx = \pi \delta^2 \Delta p$$

$$\int_0^1 p \, dx = 0 \qquad (4.40)$$

4.3.2 ANALYSIS

The equation (4.35) can be solved, with boundary conditions (4.38), for velocity u as

$$u = -1 + \frac{1}{2} \left(\frac{\partial p}{\partial x} \right) \left[I_1(r) - \frac{I_1(h)}{I_2(h)} I_2(r) \right] + (g + V_p) \frac{I_2(r)}{I_2(h)}$$

$$(4.41)$$

where we define

$$I_1(r) = \int_{h_1}^{r} \frac{r}{\mu(r)} \, dr$$

$$I_2(r) = \int_{h_1}^{r} \frac{dr}{r \mu(r)} \qquad (4.42)$$

$$I_3(r) = \int_{h_1}^{r} \frac{r^3}{\mu(r)} \, dr$$

The dimensionless flux in the wave frame $\left(q = Q' / \pi a^2 c \right)$ is obtained as

$$q = 2 \int_{h_1}^{h} ru \, dr$$

which using eqn. (4.41) gives

$$q = - \frac{1}{2}(\frac{\partial p}{\partial x})[I_3(h) - \frac{I_1^2(h)}{I_2(h)}] - (h^2 - h_1^2) + (g + V_p)[h^2 - \frac{I_1(h)}{I_2(h)}]$$

and therefore we get

$$- \frac{1}{2}(\frac{\partial p}{\partial x}) = \frac{1}{G_1}q - \frac{G_2}{G_1}(g + V_p) + \frac{h_1^2 - h_1^2}{G_1} \qquad (4.43)$$

where

$$G_1 = I_3(h) - \frac{I_1^2(h)}{I_2(h)} \qquad (4.44)$$

$$G_2 = h^2 - \frac{I_1(h)}{I_2(h)}$$

Integrating (4.43) we obtain the following relation in q and V_p

$$- \frac{1}{2} \Delta p = q F_{11} - V_p F_{12} - c_1 F_{13} + F_{14} \qquad (4.45)$$

with

$$F_{11} = \int_0^1 \frac{dx}{G_1}$$

$$F_{12} = \int_0^1 [\frac{G_2}{G_1}] dx \qquad (4.46)$$

$$F_{13} = \int_0^1 [\frac{G_2 \sin 2\pi x}{G_1}] dx$$

$$F_{14} = \int_0^1 [\frac{h^2 - h_1^2}{G_1}] dx$$

The force equilibrium conditions (4.40) provide

$$0 = F_{21}q + V_p F_{22} + c_1 F_{23} + F_{24} \qquad (4.47)$$

where

$$F_{21} = \int_0^1 \left[-G_1 \frac{I_1(h)}{I_2(h)} \right] dx$$

$$F_{22} = \int_0^1 \left[\frac{G_1 - G_2 I_1(h)}{G_1 I_2(h)} \right] dx$$

$$F_{23} = \int_0^1 \left[\frac{G_1 - G_2 I_1(h)}{G_1 I_2(h)} \right] \sin 2\pi x \, dx \qquad (4.48)$$

$$F_{24} = \int_0^1 \left[\frac{h^2 - h_1^2}{G_1 I_2(h)} \right] I_1(h) \, dx$$

The equations (4.45) and (4.47) can be solved as

$$V_p = \frac{M_{11} \Delta p + M_{12} c_1 + M_{13}}{M_{oo}} \qquad (4.49)$$

$$q = \frac{M_{21} \Delta p + M_{22} c_1 + M_{23}}{M_{oo}} \qquad (4.50)$$

where

$$M_{oo} = F_{21} F_{12} + F_{22} F_{11}$$

$$M_{11} = \frac{1}{2} F_{21}$$

$$M_{12} = -F_{21} F_{13} - F_{23} F_{11}$$

$$M_{13} = F_{21} F_{14} - F_{24} F_{11} \qquad (4.51)$$

$$M_{21} = -\frac{1}{2} F_{22}$$

$$M_{22} = F_{22} F_{13} - F_{23} F_{12}$$

$$M_{23} = -F_{22} F_{14} - F_{24} F_{12}$$

The time average flux in stationary frame can be obtained from the following expression

$$\bar{Q} = q + (1-V_p)(1 - \delta^2 + \frac{\varepsilon^2}{2} - \frac{\varepsilon_1^2}{2}) \qquad (4.52)$$

4.3.3 EFFECT OF PERIPHERAL LAYER :

To study the effect of peripheral layer, the following viscosity function has been considered

$$\mu(r) = \bar{\mu} \quad ; \quad \alpha h \leq r \leq h$$
$$= 1 \quad ; \quad 0 \leq r \leq \alpha h \qquad (4.53)$$

With this viscosity function the integrals $I_1(h), I_2(h), I_3(h)$ are evaluated as follows

$$I_1(h) = \frac{1}{2}(\alpha^2 h^2 - h_1^2) + \frac{1}{2\bar{\mu}}(h^2 - \alpha^2 h^2)$$

$$I_2(h) = \ln[(\frac{\alpha h}{h_1})(\frac{1}{\alpha})^{1/\bar{\mu}}] \qquad (4.54)$$

$$I_3(h) = \frac{1}{4}(\alpha^4 h^4 - h_1^4) + \frac{1}{4\bar{\mu}}(h^4 - \alpha^4 h^4)$$

It is evident from eqn. (4.54) that the proposal model reduces to the model discussed in section 3.3 of chapter III, when there is no peripheral layer i.e. $\alpha = 1$ or $\bar{\mu} = 1$.

4.3.4 RESULTS AND DISCUSSION

The expressions of V_p and $\bar{Q}$ have been computed from equations (4.49)-(4.54) and are graphically shown in
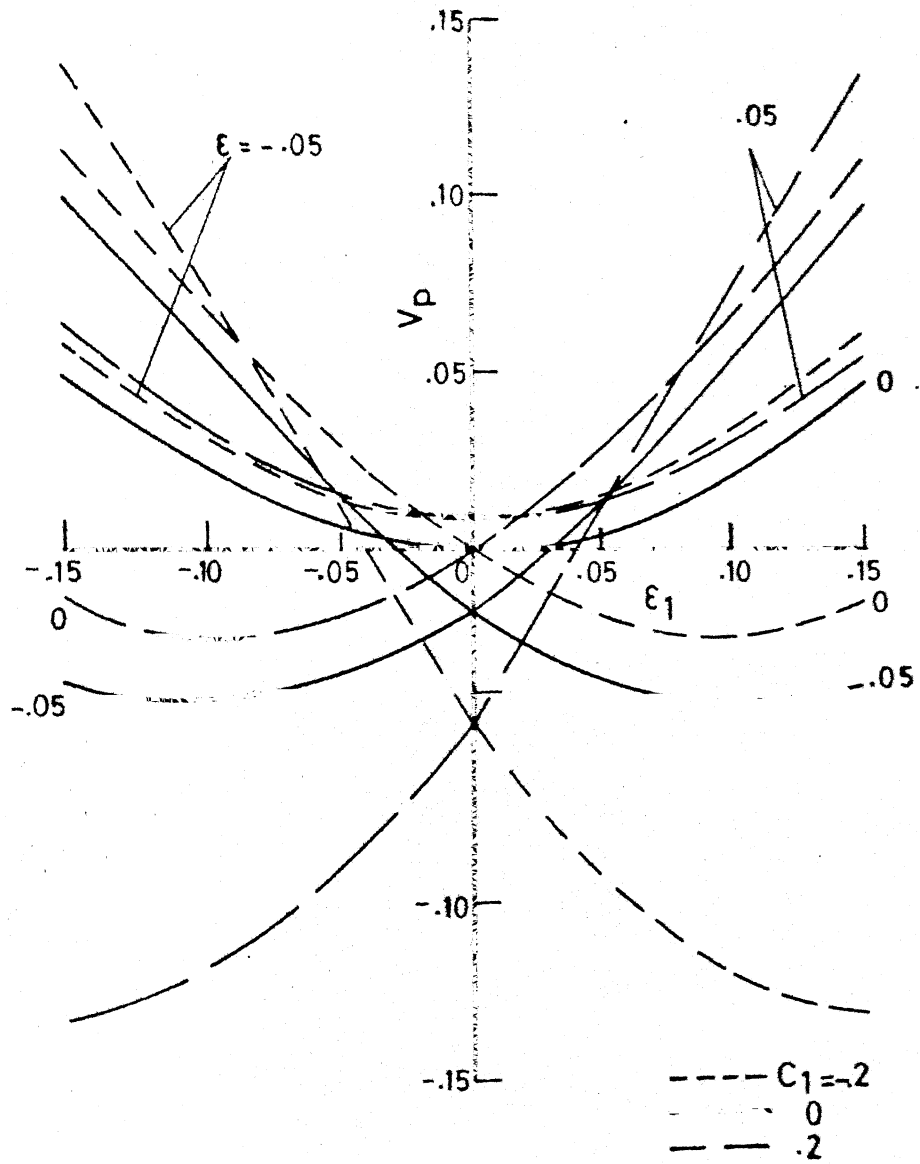
FIG. 4.9 EFFECT OF C_1 AND ε ON VARIATION OF V_P WITH ε_1 (TUBE) ($\delta = 0.2$, $\Delta p = 0.0$, $\mu = 0.75$, $\alpha = 0.0$)
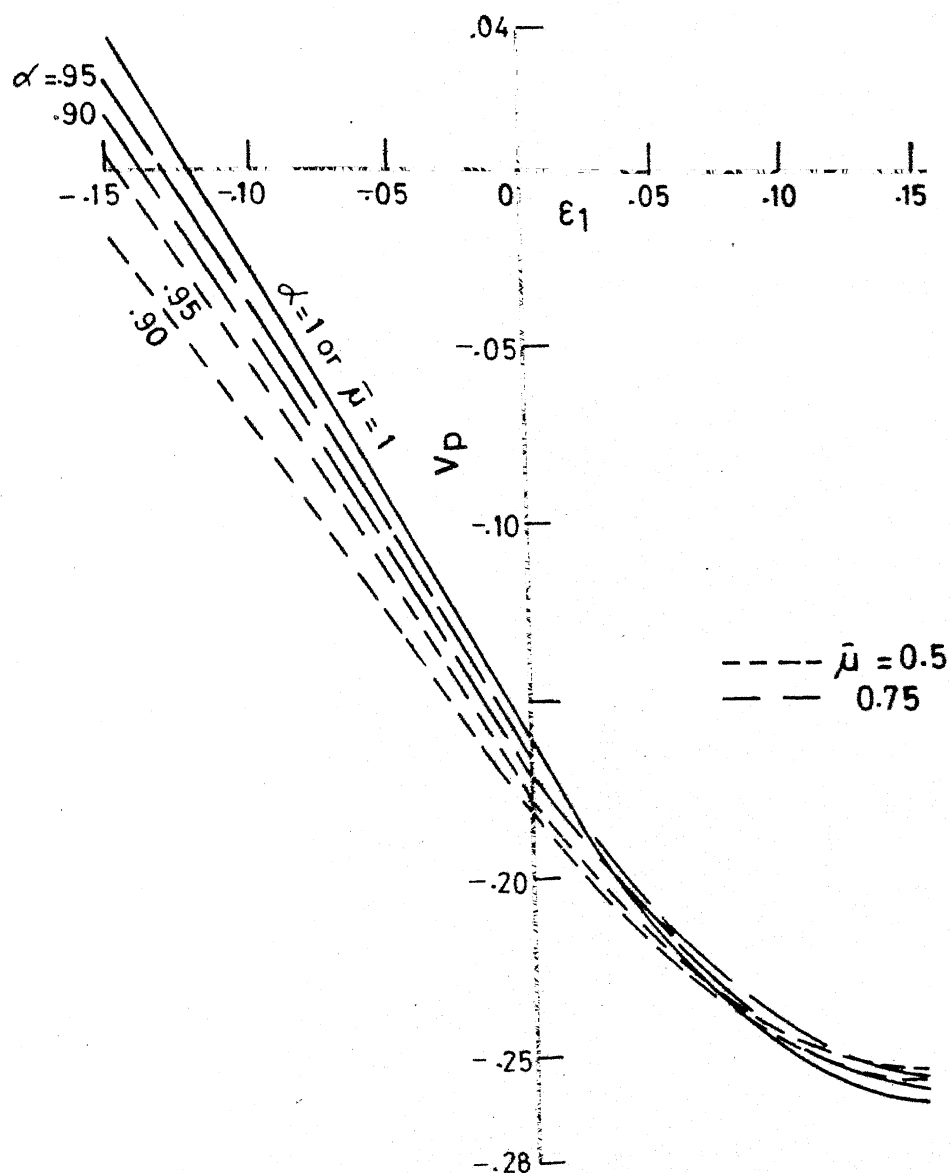
FIG. 4.10 EFFECT OF α AND $\bar{\mu}$ ON V_p (TUBE)($C_1 = -2, \varepsilon = .05,$
$\Delta p = -.5, \delta = .2$)
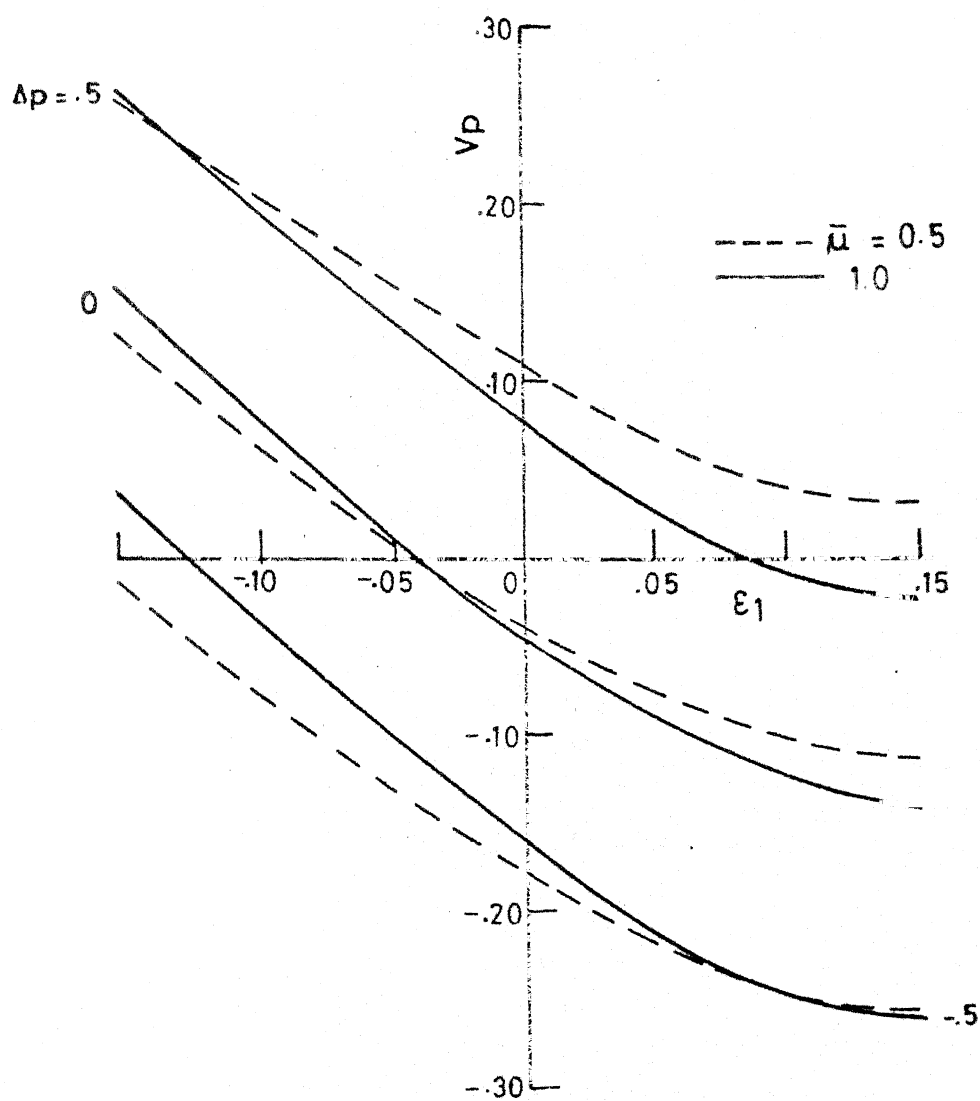
FIG. 4.11 EFFECT OF $\bar{\mu}$ AND Δp ON VARIATION OF V_p WITH ε_1 (TUBE) ($\delta = 0.2$, $C_1 = -.2$, $\varepsilon = .05$, $\alpha = .90$)

FIG. 4.12 EFFECT OF C_1 AND ε ON $\bar{Q}$ ($\delta = 0.2$, $\Delta p = 0$, $\bar{\mu} = 0.75$, $\alpha = 0.90$) (TUBE)

FIG. 4.13 EFFECT OF α AND $\bar{\mu}$ ON $\bar{Q}$ ($C_1 = -.2$, $\varepsilon = .2$, $\Delta P = -.5$, $\delta = .2$)(TUBE)
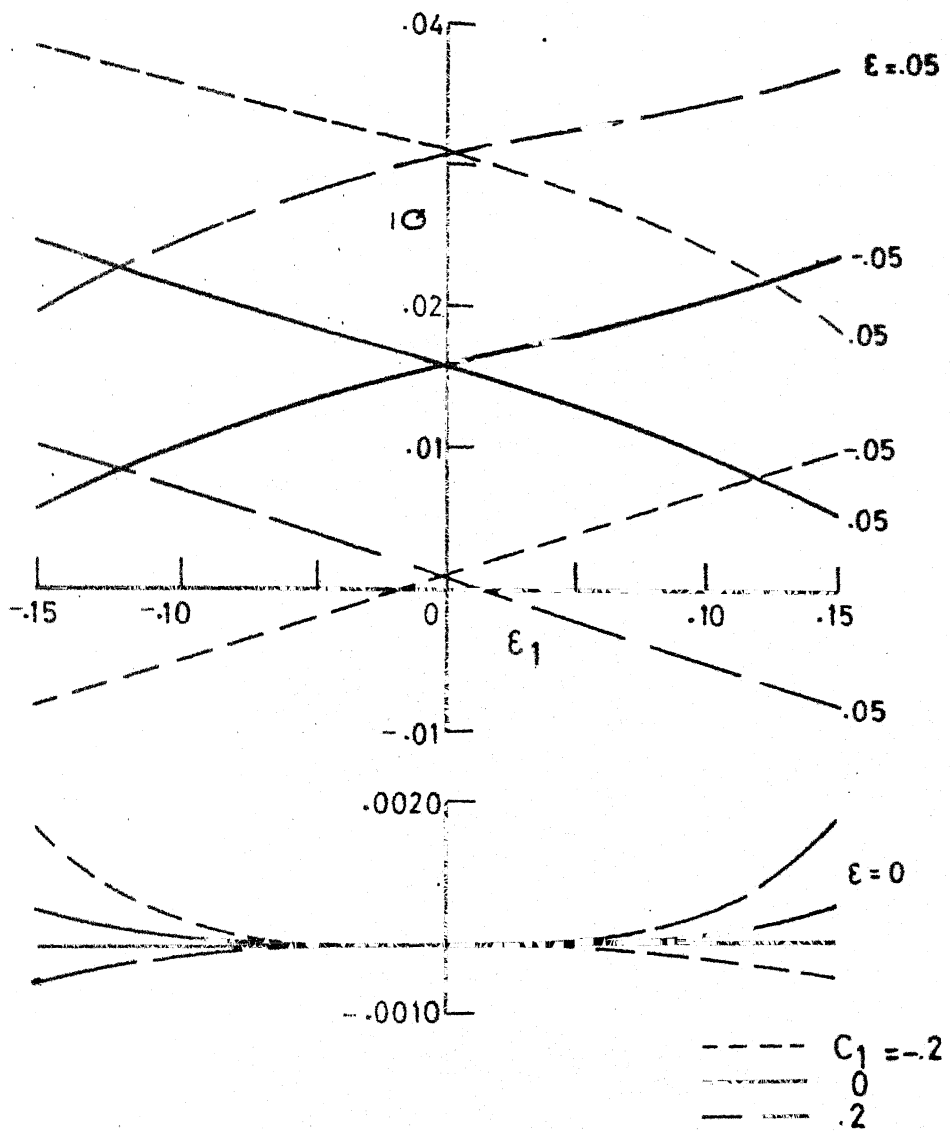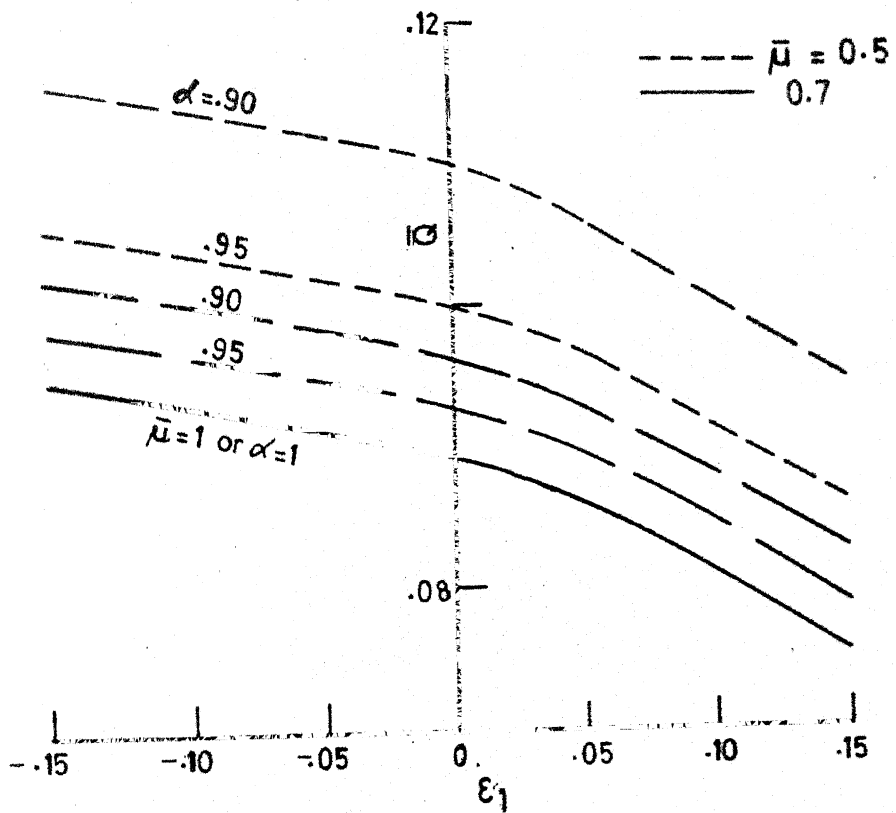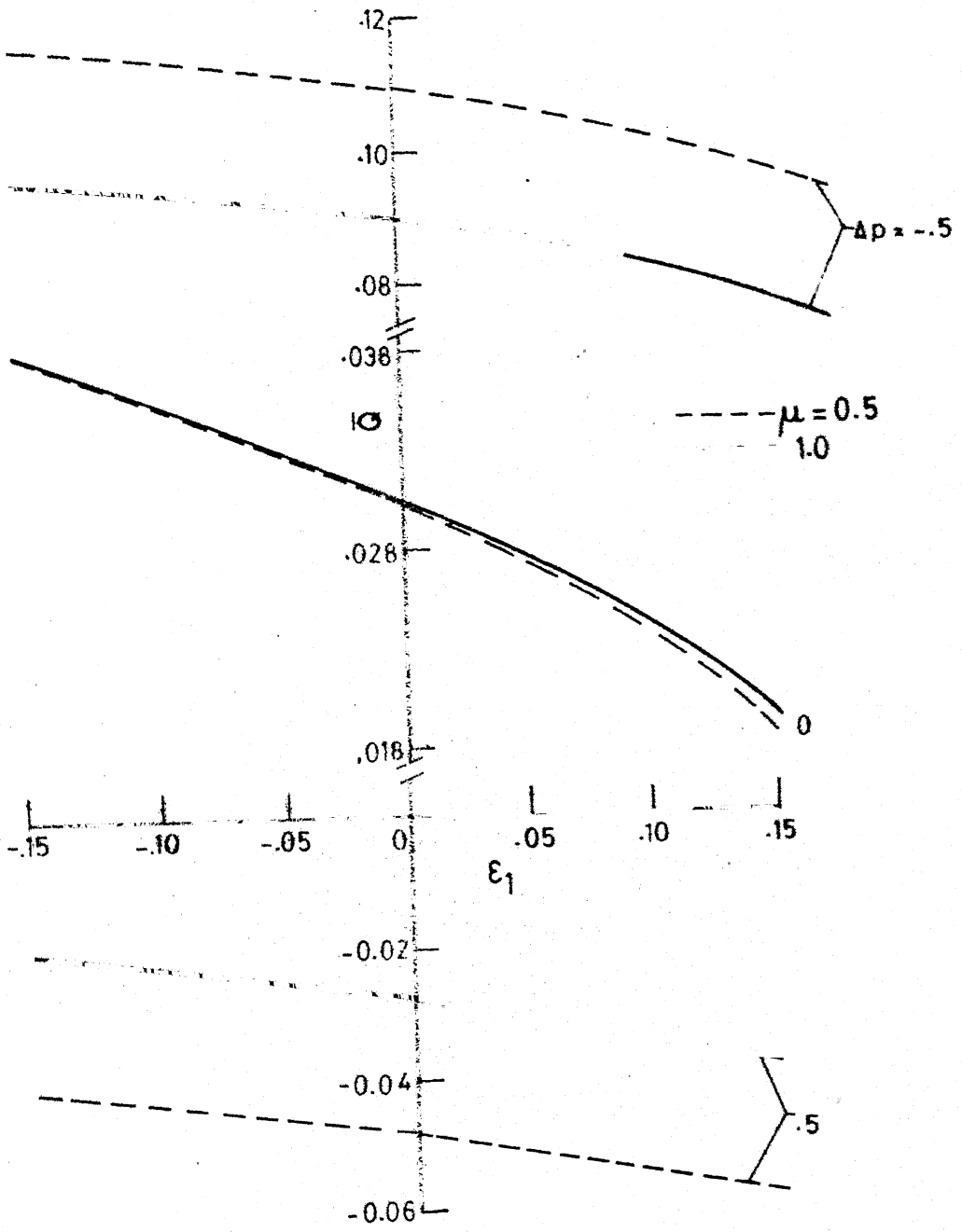
FIG. 4.14 EFFECT OF μ AND Δp ON VARIATION OF $\bar{Q}$ WITH ε_1
($\delta = 0.2$, $C_1 = -.2$, $\varepsilon = .05$, $\alpha = .90$)(TUBE)

figs. 4.9 - 4.14. Comparing the results with those
obtained in section 3.3 of chapter III, it is noticed
that for fixed viscosity, effects of ε and c_1 on V_p and
Q remain same. The effects of $\bar{\mu}$ and α are qualitatively
similar as observed in section 4.2. i.e. magnitude of
V_p and $\bar{Q}$ decrease as $\bar{\mu}$ decreases.

4.4 CONCLUSION :

Mathematical models to study the effect of peristaltic
and longitudinal motion of the walls of the transporting
duct and the viscosity variation across the cross-section
on the swimming of a micro-organism have been proposed.
It has been observed that the results obtained for the
transport in channel are qualitatively similar to those
of the tube. Apart from the results observed in chapter III,
it has also been shown that magnitude of propagation velocity
of the micro-organism decreases as the viscosity of the
peripheral layer increases.

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CHAPTER V

EFFECT OF MULTIPLE CONSTRICTIONS ON PERISTALTIC TRANSPORT

OF A FLUID THROUGH A TUBE OF NON-UNIFORM GEOMETRY

5.1 INTRODUCTION

Peristalsis is a mechanism for the transport of a
fluid achieved by the passage of progressive waves of
contraction and expansion along the walls of a distensible
tube. Many physiological fluids in the human body are
known to be transported by peristaltic action of the walls,
such as urine in ureter and chyme in intestines. It is
also speculated that peristalsis may be associated with
the vasomotion of small blood vessels [Fung and Yih (1968)].

Severals authors have contributed to the study of
peristaltic transport in both physiological and mechanical
situations [Burns and Parkes (1967), Fung and Yih (1968),
Barton and Raynor (1968), Shapiro et al. (1969), Zien and
Ostrach (1970), Manton (1975), Kaimal (1978), Shukla et al.
(1978)]. In particular Fung and Yih (1968) investigated
this problem for arbitrary wavelengths and small amplitudes.
Later, Shapiro et al. (1969) considered peristaltic
pumping in a tube under long wavelength approximation
and for very small Reynolds numbers. It has been shown
in these studies that the average flux over a time period,
is increased by increasing the amplitude of the peristaltic

wave. Li (1970) analysed peristaltic transport in circular cylindrical tube, assuming stream function as a power series, and compared the flow through tube with the flow in two dimensional channel. A detailed analysis of peristaltic pumping using finite element method has also been presented by Tong and Vawter (1973). Manton (1975) has studied this problem by perturbation techniques, using long wavelength approximation.The effect of non-Newtonian behaviour of the fluid under peristaltic motion has also been considered by many investigaters [Raju and Devanathan (1972), Acharya (1982), Shukla and Gupta (1982)].

It must be observed that in these studies the solutions have been obtained in the wave frame, where the flux is steady because of the uniform geometry of the transporting ducts. However, it is noted that cross section of various ducts in human body gradually varies along their length and therefore a steady flow through them can not be obtained even in the wave frame. Guha et al. (1975) studied the effect of peristalsis on the flow through a duct of linearly varying cross - section with particular reference to spermatic fluid transport in vas deferens. Later Gupta and Seshadri (1976) and Srivastava et al. (1983) also studied this problem and obtained the solution in the stationary frame. In these studies the value of pressure difference across a wavelength has been calculated for a given rate of flow and periodic nature of the pressure difference has been shown.

Further, it may be pointed out here that the peristaltic transport of fluid in constricted tubes has not been studied so far, though such situation may arise in intestines due to the development of polyp. The dynamical interaction between the passage of the progressive peristaltic waves and an abnormal growth in the lumen of the transporting duct may be very complex and its study may lead to a better understanding of such pathological situations. In view of this, in this chapter a mathematical model has been proposed to study the peristaltic transport of a Newtonian fluid through a circular cylindrical tube of varying cross − section with two mild local constrictions. Following Gupta and Seshadri (1976), solutions are obtained in the fixed laboratory frame under long wavelength approximation and the effect of various parameters on the pressure difference and the wall shear stress have been studied.

5.2 MATHEMATICAL ANALYSIS

Let us consider the flow of a Newtonian viscous fluid through a circular tube of non-uniform cross-section and having two constrictions along its length. The geometry of the tube, as seen in the stationary frame of reference, is shown in fig. 5.1. It is assumed that sinusoidal travelling waves are imposed on the flexible boundary of the tube. The radius of the tube, at a distance z' from
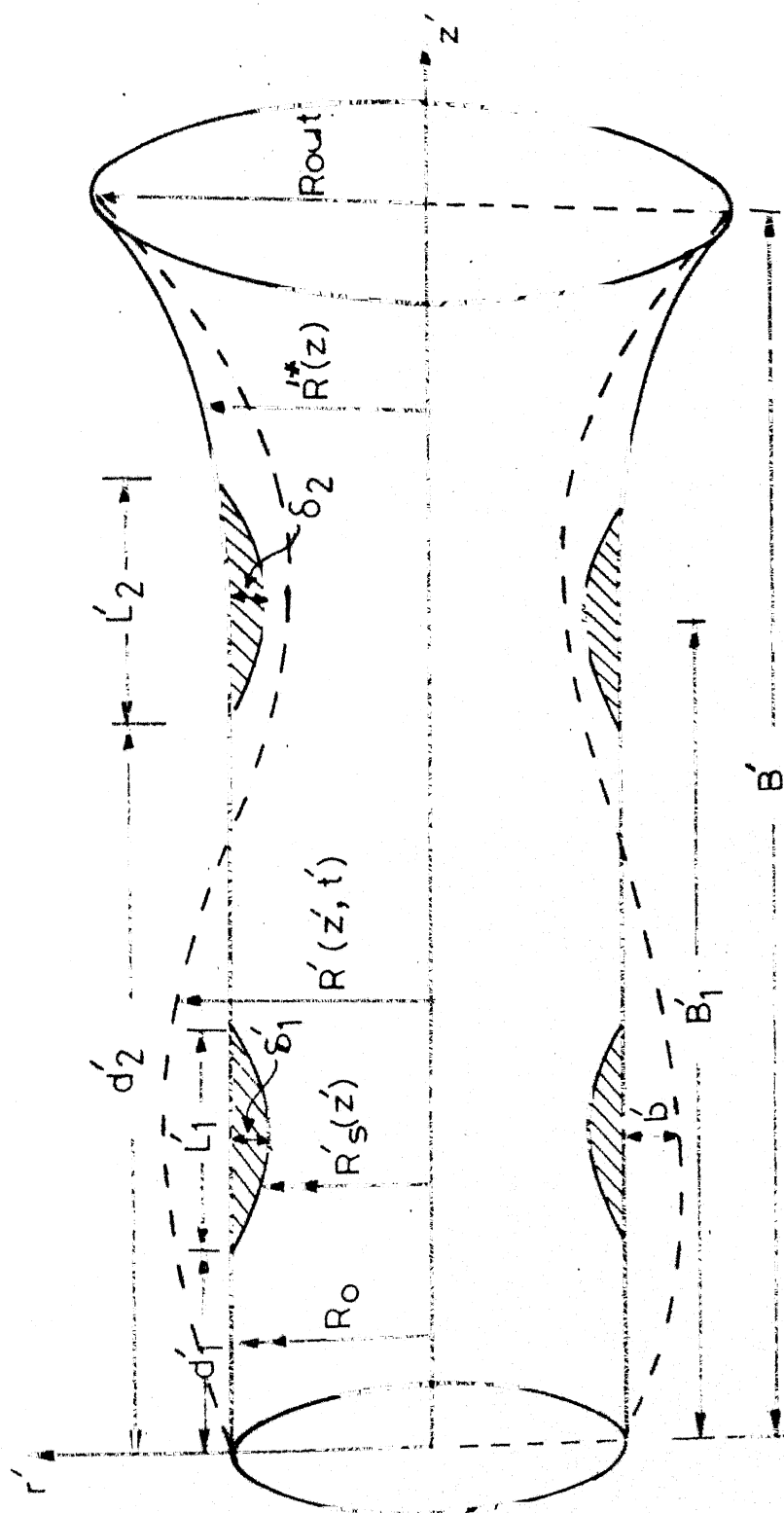
Fig.5.1 Geometry of the tube with peristalsis, multiple stenosis

origin and at instant t' is given by

$$R'(z',t') = R'_s(z') + b' \sin \frac{2\pi}{\lambda}(z'-ct') \qquad (5.1)$$

where b', λ and c are the amplitude, wavelength and speed of the peristaltic wave and $R'_s(z')$ is the variable radius of the lumen of the tube when there is no peristalsis.

The constrictions are assumed in series, mild and are symmetric about axis and therefore the lumen of the tube can be assumed to be given by

$$R'_s(z') = \begin{cases} R'_o & 0 \le z' \le d'_1 \\[2mm] R'_o - \dfrac{\delta'_1}{2}[1 + \cos \dfrac{2\pi}{L'_1}(z'-d'_1-\dfrac{L'_1}{2})]; & d'_1 \le z' \le d'_1 \le L'_1 \\[2mm] R'_o & ; d'_1+L'_1 \le z' \le B'_1 - \dfrac{L'_2}{2} \\[2mm] R'_o - \dfrac{\delta'_2}{2}[1 + \cos \dfrac{2\pi}{L'_2}(z'-B'_1)] & ; B'_1 - \dfrac{L'_2}{2} \le z' \le B'_1 \\[2mm] R^{*'}(z') - \dfrac{\delta'_2}{2}[1 + \cos \dfrac{2\pi}{L'_2}(z'-B'_1)]; & B'_1 \le z' \le B'_1 + \dfrac{L'_2}{2} \\[2mm] R^{*'}(z') & ; B'_1 + \dfrac{L'_2}{2} \le z' \le B' \end{cases}$$

$$(5.2)$$

As shown in fig. (5.1), the constrictions are in the regions $d'_1 \le z' \le d'_1+L'_1$ and $B'_1-L'_2/2 \le z' \le B'_1+L'_2/2$ respectively. R'_o is the constant radius of the unstenosed part of the tube in the region

$d'_1 \le z' \le B'_1$ and $R^{*'}(z')$ is gradually varying radius in the region $B'_1 \le z' \le B'$. L'_i and δ'_i are the lengths and the maximum thickness of the two mild stenoses (the suffixes 1 and 2 indicate the first and , second stenosis respectively) and

are such that they follow the restrictions for mild stenosis [Young (1968)] i.e,

$$\lceil \delta'_i << \min (R'_o, R'_{out})$$

$$(i = 1,2) \qquad (5.3)$$

$$\lfloor \delta'_i << L'_i$$

where $R_{out} = R'_s(B')$.

The flow is considered to be inertia free, laminar and axially-symmetric. Following Young (1968) and Shukla et al. (1980) the basic equations governing the flow are

$$-\frac{\partial p'}{\partial z'} + \frac{\mu}{r'}\frac{\partial}{\partial r'}(r'\frac{\partial w'}{\partial r'}) = 0 \qquad (5.4)$$

$$\frac{\partial w'}{\partial z'} + \frac{1}{r'}\frac{\partial}{\partial r'}(r'u') = 0 \qquad (5.5)$$

where μ is the viscosity of the fluid, p' is the pressure and u' and w' are the velocity components of the fluid in the radial and axial directions respectively.

The boundary conditions are

$$w' = 0 \qquad \text{at } r' = R'(z',t')$$

$$\qquad (5.6)$$

$$\frac{\partial w'}{\partial r'} = 0 \qquad \text{at } r' = 0$$

Using the following non-dimensional quantities

$$t = ct'/\lambda, \quad w = w'/c, \quad p = p'R_o^2/\mu c\lambda$$

$$(z,d_1,L_1,L_2,B_1) = (z',d'_1,L'_1,L'_2,B'_1)/B' \qquad (5.7)$$

$$(r,R,R_s,R^*,\varepsilon,\delta_i) = (r',R',R'_s,R^{*'},b',\delta'_i)/R'_o$$

and solving equations (5.5) and (5.6) we have

$$w = - \frac{1}{4}(\frac{\partial p}{\partial z})(R^2 - r^2)$$ (5.8)

The instantaneous volumetric flow rate, in dimensionless form can be written as

$$Q(z,t) = 2\int_0^R r\, w\, dr = - \frac{[R(z,t)]^4}{8}(\frac{\partial p}{\partial z})$$

which gives

$$\frac{\partial p}{\partial z} = -8 \frac{Q(z,t)}{[R(z,t)]^4}$$ (5.9)

The eqn. (5.9) can be integrated with respect to z, over a wavelength to give instantaneous pressure rise over a wave length $\Delta p(t)$ as

$$\Delta p(t) = -8 \int_0^1 \frac{Q(z,t)}{[R(z,t)]^4}\, dz$$ (5.10)

The pressure rise over a wavelength has been calculated for a given form of $Q(z,t)$ as the inverse problem is difficult due to non-availability of analytical solution of the integral involved.

It is evident from eqn. (5.10) that $\Delta p(t)$ will always be negative for a constant $Q(z,t)$ implying no pumping action. Therefore $Q(z,t)$ has been assumed to be a function of $(z-t)$ of the following form [Gupta and Seshadri (1976)]

$$Q(z,t) = \bar{Q} + [R^2(z,t) - R_s^2(z,t)] - \frac{1}{2}\epsilon^2$$ (5.11)

where $\bar{Q}$ is the constant time mean flow over one period of the wave. It should be noted here that the flux in wave frame $q(z,t)$, as defined by Jaffrin and Shapiro (1971), will be a function of z in this case.

The equation (5.10), therefore, becomes

$$\Delta p(t) = -8(\bar{Q} - \tfrac{1}{2}\epsilon^2) \int_o^1 \frac{dz}{[R(z,t)]^4} + 8 \int_o^1 \frac{R_s^2(z)}{[R(z,t)]^4} dz$$

$$-8 \int_o^1 \frac{dz}{[R(z,t)]^2} \qquad (5.12)$$

which gives the pressure rise $\Delta p(t)$ in terms of the average volumetric flow $\bar{Q}$.

The instantaneous shearing stress $\tau_w(z,t)$ on the wall of the tube is given by [Manton (1971)]

$$\tau_w(z,t) = D(z,t)\left[r \frac{\partial w}{\partial r}\right]_{r = R(z,t)} \qquad (5.13)$$

where

$$D(z,t) = \frac{1 - \beta^2 (\frac{dR}{dz})^2}{1 + \beta^2 (\frac{dR}{dz})^2} \qquad (5.14)$$

and $\beta = R_o'/B'$. $\qquad (5.15)$

Integrating (5.15) over a time period and using (5.14), (5.15) alongwith the form of $Q(z,t)$ from (5.11) we get the time-averaged shear stress $\tau(z)$ on the wall of the tube as

$$\tau(z) = -4(\bar{Q} - \tfrac{1}{2}\epsilon^2 - R_s^2(z)) \int_o^1 \frac{D(z,t)}{[R(z,t)]^2} dt - 4 \int_o^1 D(z,t)dt \qquad (5.16)$$

5.3 RESULTS AND DISCUSSIONS :

To explicity see the effects of various parameters
on the pressure rise and the shear stress, the expressions
for Δp and $\tau_w(z,t)$ on the second constriction ((5.12),(5.16))
have been numerically calculated on DEC-1090. The integrals
involved , are computed using trapezoidal rule which provided
most accurate results (accuracy upto 10^{-6}) in comparison
to some other quadrature formulae. To study the effect
of varying cross-section, $R^*(z)$ has been taken as

$$R^*(z) = \exp.\left[K(z-B_1)^2\right] \qquad (5.17)$$

where the parameter K determines the convergence/divergence
of the tube, the negative, zero and positive values of K
correspond to converging uniform and diverging tube
respectively.

The following values of the parameters regarding the
constrictions have been chosen in the calculation for Δp
and τ

$$d_1 = 0.2, \ L_1 = 0.2, \ L_2 = 0.2, \ B_1 = 0.8$$

and $\beta = 0.1$.

Values of Δp given by eqn. (5.12) are plotted as a function
of time t for various values of $\bar{Q}$, K and δ_1, δ_2 in (fig.5.2-
5.4). For fixed values of δ_1 and δ_2 , the effect of $\bar{Q}$
and K is shown for $\varepsilon = 0.4$ in fig. 5.2 and $\varepsilon = 0.5$ in
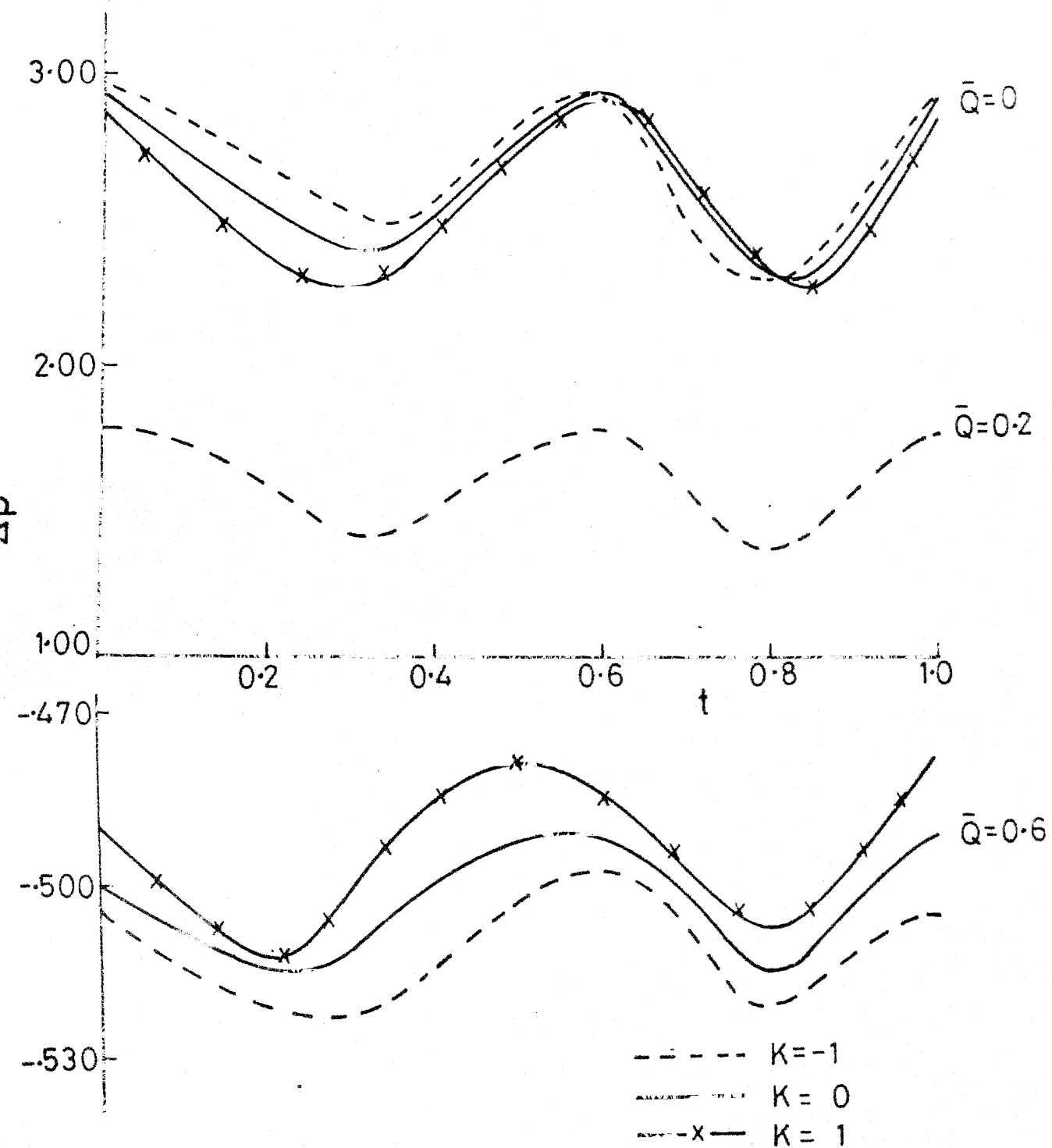
Fig. 5·2 Effect of K and $\bar{Q}$ on variation of Δp with t $(\mathcal{E} = \cdot4, \delta_1 = \cdot1, \delta_2 = \cdot1)$
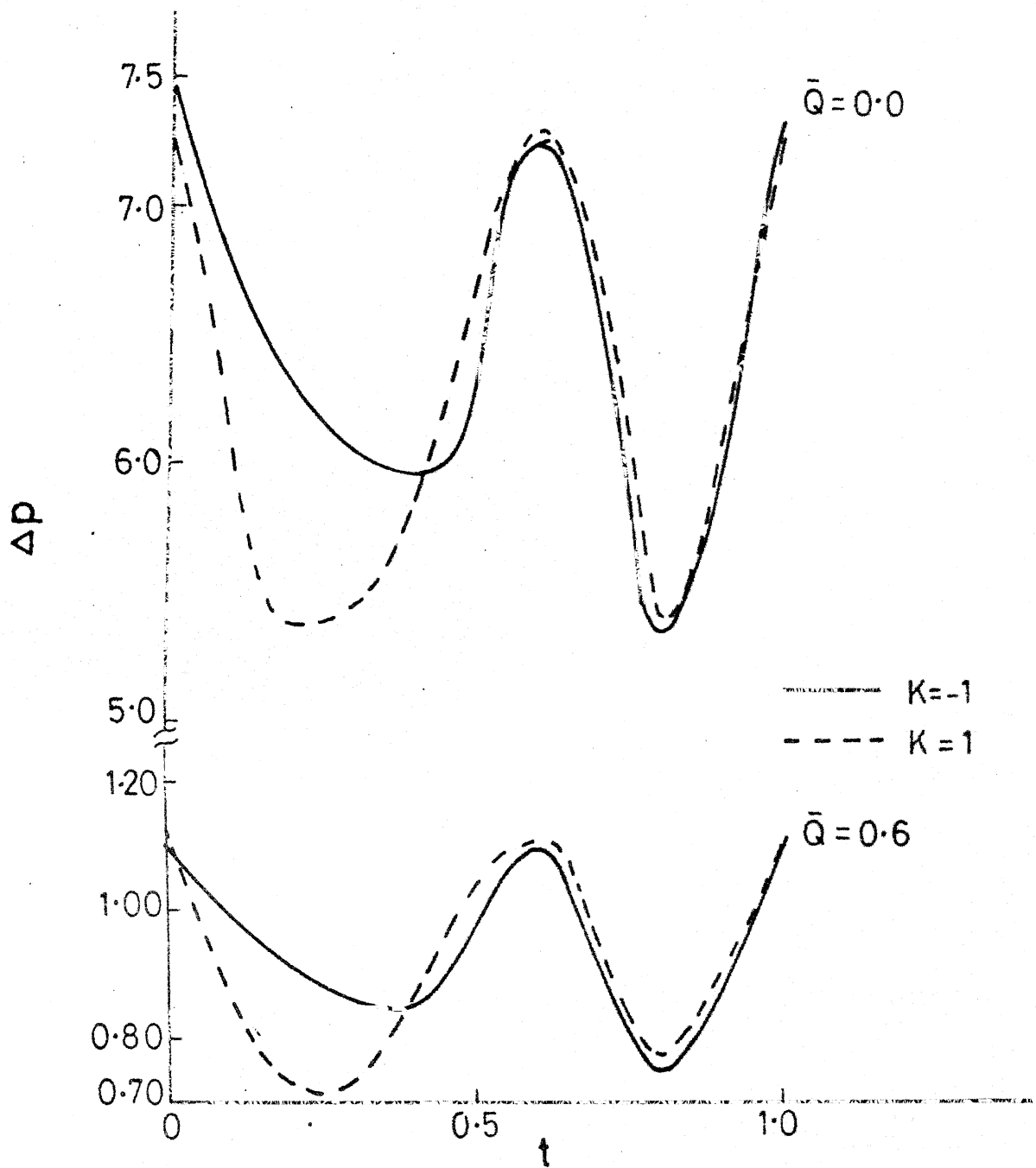
Fig.5·3 Effect of K and Q on variation of
with t (ε=·5 δ₁=·1 δ₂=·1)
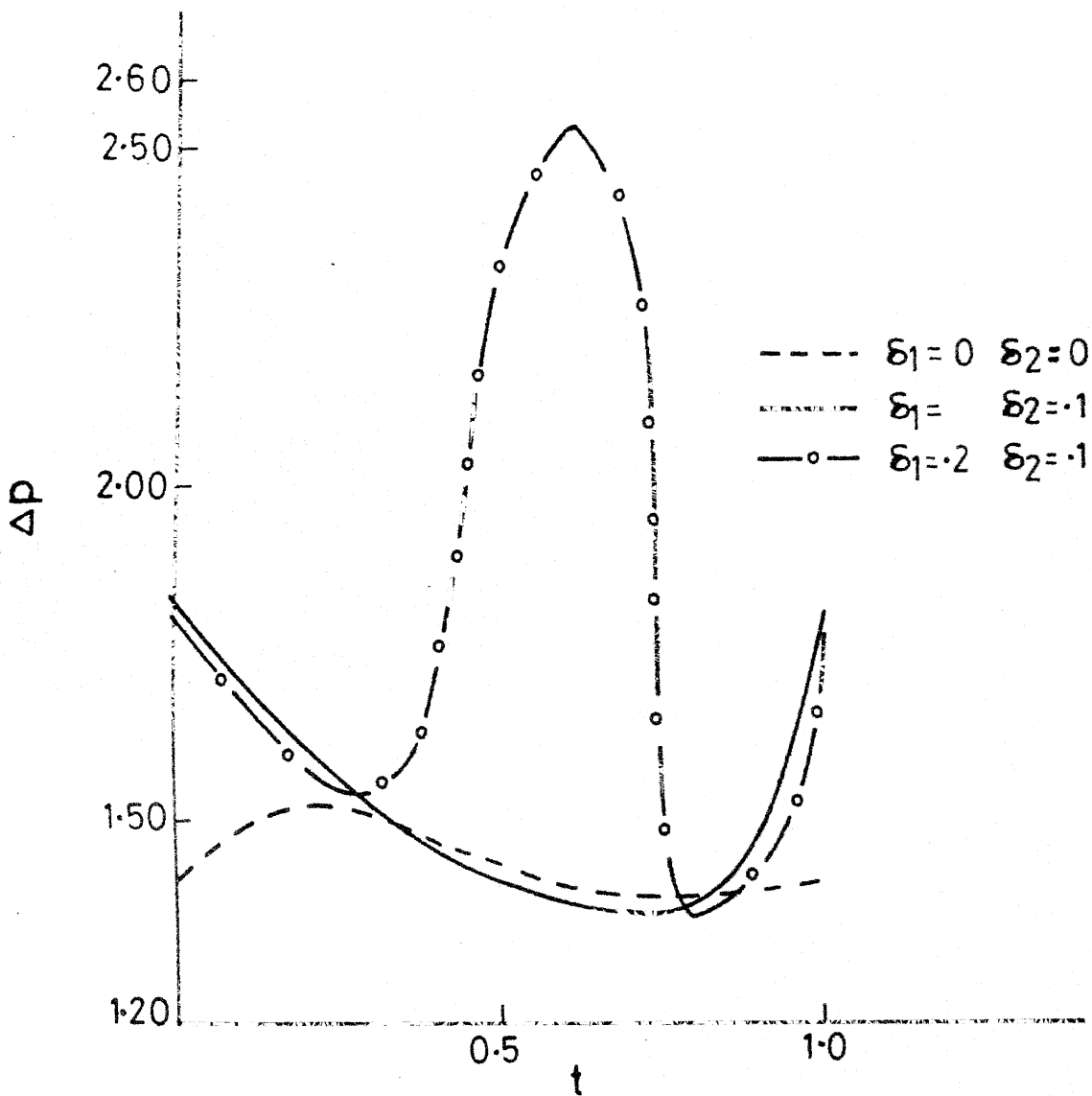
Fig.5·4 Effect of δ_1 and δ_1 on variation of
with t ($\varepsilon = ·4$, $\bar{Q} = 2$, $K = 0$)

fig. 5.3, while for fixed values of $\bar{Q}$, K and ε, the effect of δ_1 and δ_2 is shown in fig.(5.4).

For fixed values of $\tilde{Q}$, K , ε , δ_1 and δ_2, oscillatory behaviour of t with respect to t can be observed (fig.5.2,5.3). As t increases Δp goes down, attains a minimum at about 1/3rd of the time period of peristaltic wave ; it attains a local maximum at t, slightly greater than half time period and the second local minimum is obtained about 3/4th of time period. It must be noted that this behaviour of two-plus-one extrema i.e. one maximum between two local minima is due to presence of two constrictions and is absent in the results obtained by Gupta and Seshadri (1976). Effect of increasing the flux is to decrease pressure rise $\Delta p(\bar{Q} = 0$ and 0.2) and it becomes negative for $\bar{Q} = 0.6$ (see.fig. 5.2). Also comparing figs. (5.2) and (5.3) it can be noted that Δp increases with increase in ε and decrease in $\bar{Q}$. Effect of increasing K is to decrease the magnitude of Δp. Comparing with the results obtained by Gupta and Seshadri(1976) it can be observed that (-)ve values of Δp can be obtained at a lesser value of $\bar{Q}$ in the proposed model and this effect may be due to the presence of constrictions.

Fig. 5.4 shows the effect of δ_1 and δ_2 on Δp. In absence of any constriction ($\delta_1 = 0$, $\delta_2 = 0$) the pressure attains a maximum and then reaches its minimum value. For

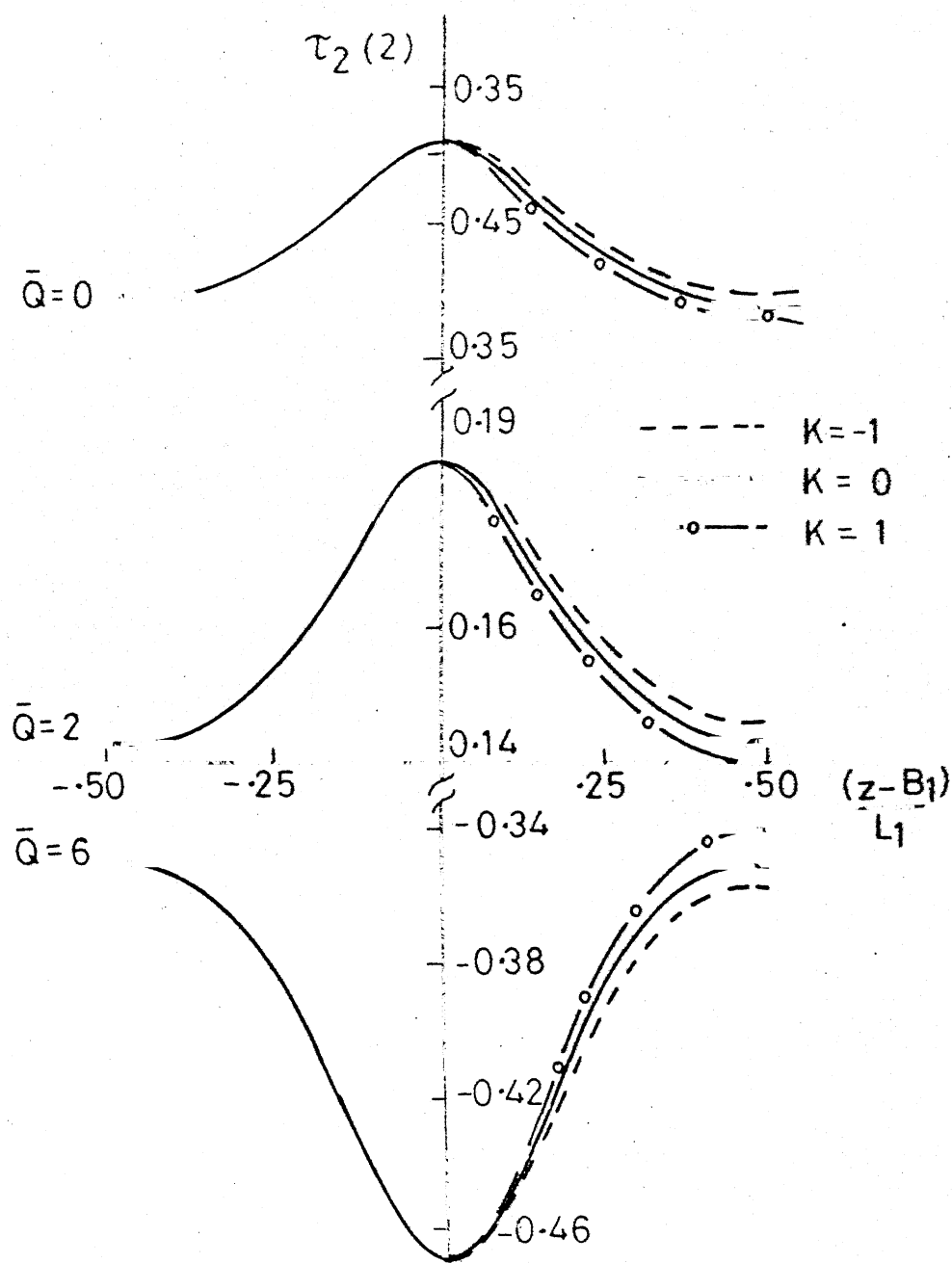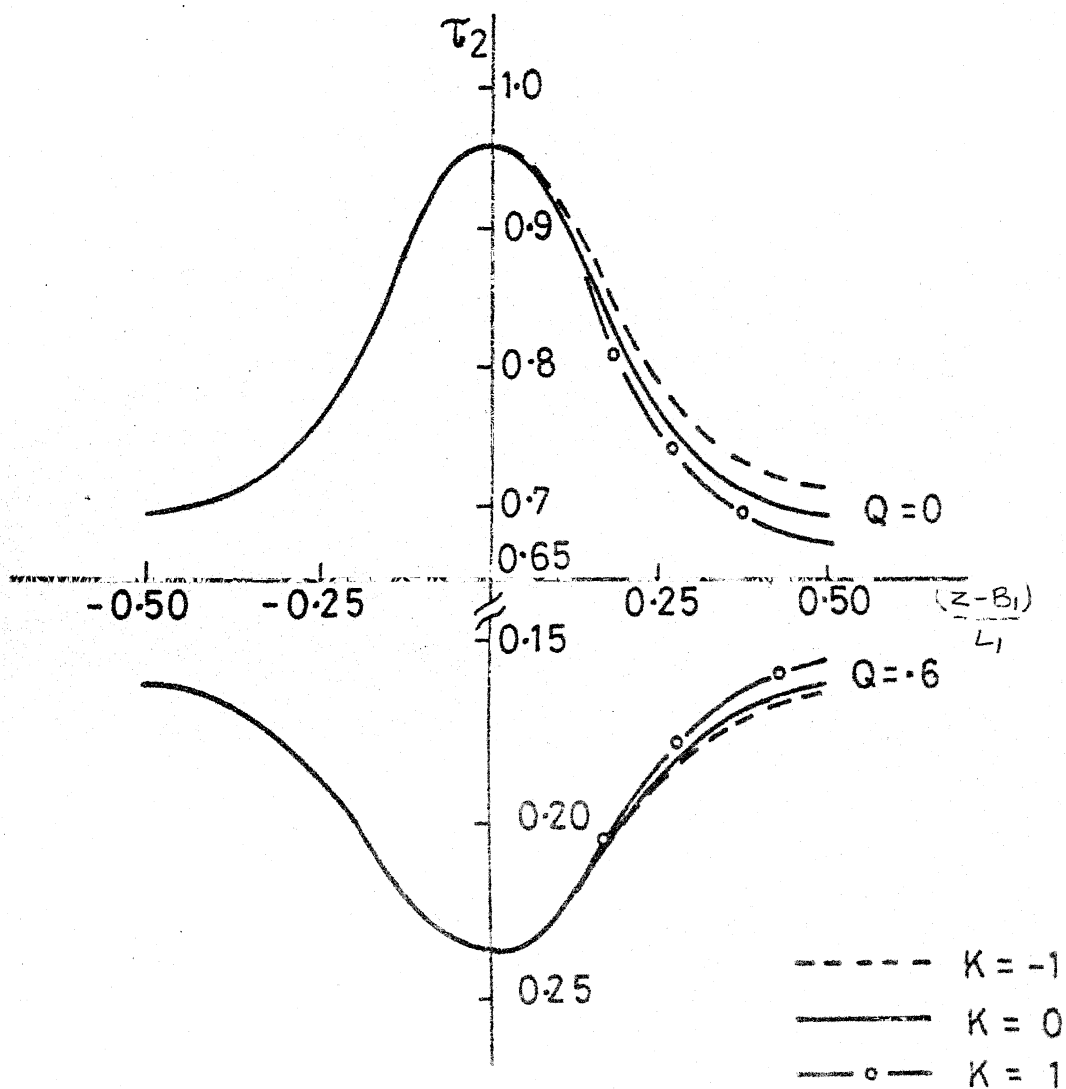Fig.5.5 Effect of K and $\bar{Q}$ on $\tau_2(2)$ ($\varepsilon = \cdot 4$, $\delta_2 = 1$)

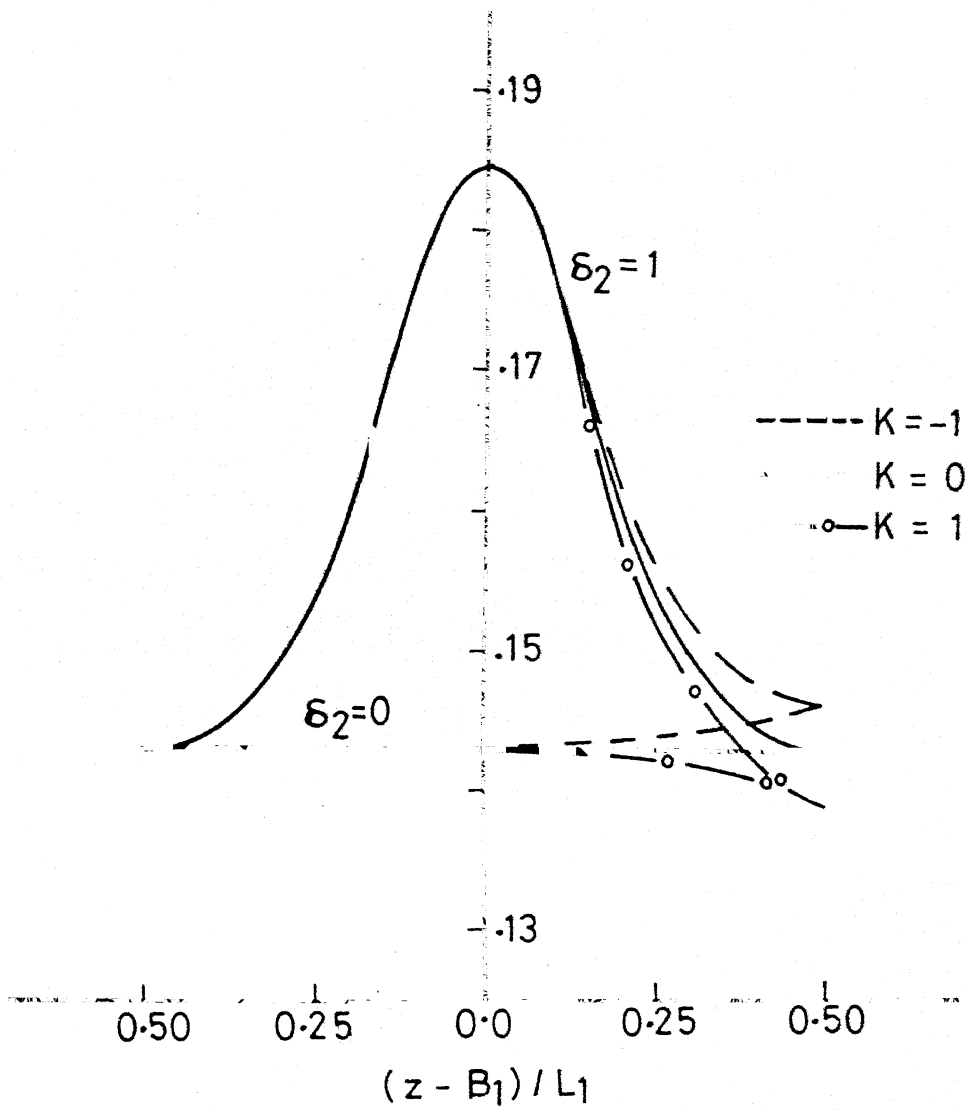Fig.5·6 Effect of Q and K on τ_2 ($\varepsilon = \cdot5$ $\delta_2 = 1$)

Fig.5·7 Effect of δ_2 on τ_2 ($\varepsilon = 0.4$ $\bar{Q} = 0.2$)

a single constriction $(\delta_1 = 0, \delta_2 = .1)$ the local maximum
disappears from the given range. One maximum and two
minima are observed when both constrictions are present
$(\delta_1 = .1, \delta_2 = .1)$, the maximum being much higher than any
other value.

Effects of $\bar{Q}$ and K are shown, in figs. 5.5 ($\varepsilon = .4$)
and fig. 5.6 ($\varepsilon = .5$), on τ_2, the stress on the second
constriction ($|z - B_1| \leq L_2$). The maximum value of τ_2 is
observed at the point of maximum thickness of the constriction.
The magnitude of τ_2 decreases as $\bar{Q}$ increases and increases
with ε and δ_2.

5.4 CONCLUSION :

A model has been presented to study the peristaltic
transport of a Newtonian fluid through a circular tube
of varying cross section and having two constrictions.
Following Gupta and Seshadri (1976), the solutions have
been obtained in the laboratory frame and it has been shown
that the pressure rise over a wavelength and the shear
stress on constrictions increase as the amplitude of the
peristaltic wave and the thickness of the constrictions
are increased. This result can be applied to the case of
intestines having abnormal constrictions.

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CHAPTER VI

FLOW OF NON-NEWTONIAN FLUID THROUGH A TUBE WITH

MULTIPLE STENOSES

6.1 INTRODUCTION :

It is well known that stenosis, which refers to abnormal growth in the lumen of artery, is a result of an arterial desease. It is caused mainly due to intravascular atherosclerotic plaques which develop at the arterial wall, thus occluding the lumen of the artery partly or fully. Although the etiology of the initiation of the stenosis is not clearly understood, it has been realized that hydrodynamic factors such as wall stress, pressure distribution and seperation phenomenon play an important role in the development and progression of this desease [Rodkiwitcz (1977) and Stehbens (1982)]. Hence considerable attention has been paid to the theoretical and experimental studies of blood flow in stenotic region under different conditions [Young (1968), Caro et al. (1971), Nerem (1974), Deshpande et.al.(1976), Mc Donald (1979), Shukla et.al.(1980a), Padmanabhan and Devanathan (1981) and Talukder et al.(1976)]. Most of these analytical studies have investigated the flow characteristics of blood in arteries with mild/non-critical stenosis by considering blood as a Newtonian Fluid [Young (1979)]. However it may be pointed out that blood being a suspension of cells in plasma, has been found to behave as a non-Newtonian fluid at low shears rates

in tubes of small diameters. In view of this some investigators
have considered non-Newtonian behaviour of blood [Shukla et al.
(1980b and c), Williams and Javadpour (1980), Devanathan and
Parvathamma (1983)]. Shukla et al. (1980b) studied the flow
characteristics for mild stenosis by considering a non-Newtonian
power law fluid model for blood. They have also considered the
effect of peripheral layer on the flow of blood to take into
account the clustering of cells in the central zone [Shukla
et al. (1980c)].

In above mentioned studies only the effect of single
stenosis has been considered and the tube was taken to be of
uniform cross-section. However it may be noted that many of the
blood vessels change their cross-section slowly along their
lengths [Schneck et al. (1975)] and may have multiple stenoses
at junctions, bends etc. Talukder et al. (1976) have conducted
experiments in the case of blood flow in artery with multiple
stenoses and pointed out that the pressure drop increases
linearly with numbers of stenoses. Keeping this in view, in
this chapter a model for the flow of a power law fluid through
tubes of non-uniform cross-section with two stenoses is investi-
gated. The non-uniform distribution of the red cells across
the tube and the existence of the peripheral layer have been
accounted for by considering radial variation of the consistency.
Solutions for mild stenoses have been obtained and the effects
of heights of stenoses, flow behaviour index etc. on the
resistance to flow and wall shear stress have been studied. It

has been shown that the model presented here, predicts the results in conformity with the experimental work of Talukder et al. (1976).

6.2 ANALYSIS :

Consider the flow of blood (non-Newtonian power law fluid) through an artery (circular tube) with two stenoses, the non-stenosed parts of which consist of regions with uniform radius R_o and slowly varying radius $R^*(z)$ as shown in Fig. 6.1. It is assumed that the stenoses have developed symmetrically and are very mild, the shapes of which can be represented by cosine curves [Young (1968)]. Thus, the varying radius $R(z)$ of the tube can be written as follows :

$$R(z) = \begin{cases} R_o & ; \ 0 \leq z \leq d_1 \\ R_o - \frac{\delta_1}{L_1} [1+\cos\frac{2\pi}{L_1} (z-d_1-\frac{L_1}{2})] & ; \ d_1 \leq z \leq d_1+L_1 \\ R_o & ; \ d_1+L_1 \leq z \leq B_1 - \frac{L_2}{2} \\ R_o - \frac{\delta_2}{2} [1+\cos\frac{2\pi}{L_2}(z-B_1)] & ; \ B_1 - \frac{L_2}{2} \leq z \leq B_1 \\ R^*(z) - \frac{\delta_2}{2} [1+\cos\frac{2\pi}{L_2}(z-B_1)] & ; \ B_1 \leq z \leq B_1+\frac{L_2}{2} \\ R^*(z). & ; \ B_1 + \frac{L_2}{2} \leq z \leq B \end{cases}$$

$$(6.1)$$

Here, L_i and δ_i (i = 1,2) are the lengths and the maximum thickness of the two stenoses (the suffixes 1 and 2 here indicate the first and second stenosis respectively) and are such that the restrictions for mild stenosis [Young (1968)] are satisfied.
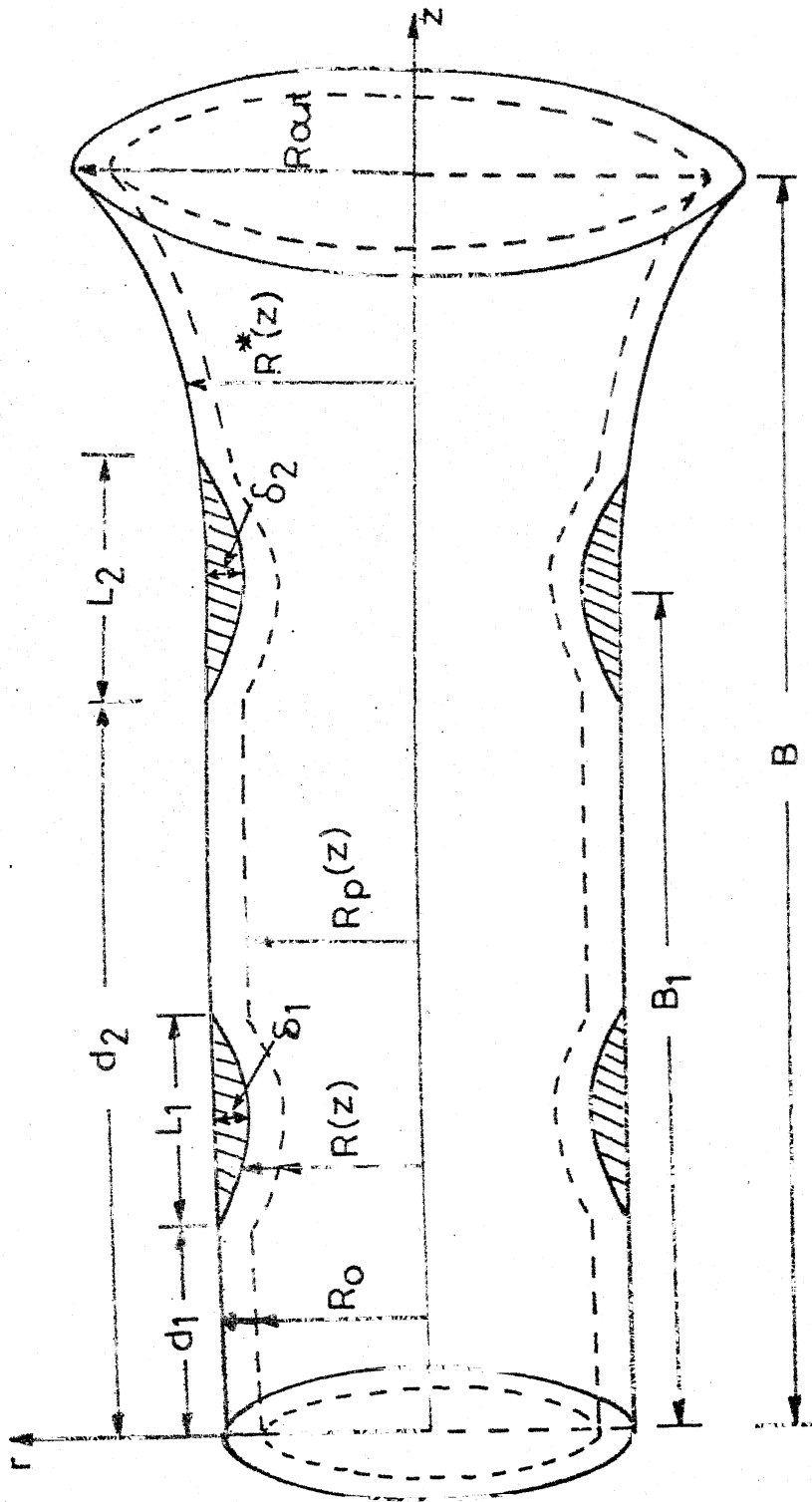
Fig.6·1 Geometry of the tube with multiple stenosis

$$\begin{cases} \delta_i \ll \min (R_o, R_{out}) \\ \delta_i \ll L_i \end{cases} \qquad (i = 1, 2) \qquad (6.2)$$

where $R_{out} = R(z)$ at $z = B$.

Following Young (1968) and Shukla et al. (1980b) we consider the flow to be steady, laminar and axially symmetric. As in this analysis blood has been characterized by the power-law model whose consistency may vary along the radial direction, the basic equation governing the flow is

$$\frac{1}{r} \frac{\partial}{\partial r}(r\tau) + \frac{dp}{dz} = 0 \qquad (6.3)$$

where the shear stress τ is given by

$$\tau = -m(r) \, | \frac{\partial w}{\partial r} |^{n-1} \, \frac{\partial w}{\partial r} = m(r)(- \frac{\partial w}{\partial r})^n,$$

$$\frac{\partial w}{\partial r} < 0, \quad R > r > 0 \qquad (6.4)$$

Here w is the axial velocity, p the pressure, n is power-law exponent which is assumed to be constant and may be taken as the mean of the flow behaviour index over the radius, $m(r)$ is the consistency function which varies radially.

Combining eqns. (6.3) and (6.4) we get our basic equation as

$$\frac{1}{r} \frac{\partial}{\partial r} \left[r \, m(r) (- \frac{\partial w}{\partial r})^n \right] + \frac{dp}{dz} = 0 \qquad (6.5)$$

The boundary conditions for w are provided by the no slip condition at the boundary of the tube and the axial symmetry

of the flow

$$\frac{\partial w}{\partial r} = 0 \quad \text{at } r = 0$$

$$w = 0 \quad \text{at } r = R(z)$$

(6.6)

Solving equation (6.5) and using the conditions (6.6) the axial velocity can be obtained as

$$w = (- \frac{1}{2} \frac{dp}{dz})^{1/n} \int_{r}^{R(z)} \left[\frac{r}{m(r)} \right]^{1/n} dr$$

(6.7)

The volumetric flow rate is defined as

$$Q = \int_{0}^{R(z)} 2\pi r w \, dr$$

(6.8)

which on substituting w from eqn. (6.7) into eqn. (6.8) is given by

$$\frac{dp}{dz} = -2 \left[\frac{Q}{\pi} \frac{1}{I(z)} \right]^{n}$$

(6.9)

where

$$I(z) = \int_{0}^{R(z)} \frac{r^{2+1/n}}{[m(r)]^{1/n}} dr$$

(6.10)

Integrating eqn. (6.9) with respect to z we get the pressure difference Δp along the total length of the tube as follows

$$\Delta p = 2 (\frac{Q}{\pi})^{n} \int_{0}^{B} \frac{dz}{(I(z)]^{n}}$$

(6.11)

The resistance to the flow λ is calculated from (6.11) as follows

$$\lambda = \frac{\Delta p}{Q} = \frac{2}{Q} (\frac{Q}{\pi})^{n} \int_{0}^{B} \frac{dz}{[I(z)]^{n}}$$

(6.12)

The shearing stress τ_{w} on the wall of the tube $(r = R(z))$

is given by [Manton (1971)]

$$\tau_w = \frac{1-(dR/dz)^2}{1+(dR/dz)^2} \left[m(r) \left(-\frac{\partial w}{\partial r} \right)^n \right]_{r=R(z)} \qquad (6.13)$$

which on using eqns. (6.7) and (6.9) can be obtained as

$$\tau_w = \frac{1-(dR/dz)^2}{1+(dR/dz)^2} \left[R(z) \left\{ \frac{Q}{\pi I(z)} \right\}^n \right] \qquad (6.14)$$

Let λ_N and τ_N be the resistance to the flow and the shearing stress at the wall respectively for the flow of a power law fluid (with fluid behaviour index n and constant consistency m_1) with flux Q, in a tube of uniform radius R_o and length B having no stenosis. These can be obtained from (6.12) and (6.14) respectively as

$$\lambda_N = \frac{2}{Q} \left(\frac{Q}{\pi} \right)^n \left(\frac{3n+1}{n} \right)^n \frac{m_1 B}{R_o^{3n+1}} \qquad (6.15)$$

and

$$\tau_N = \left(\frac{Q}{\pi} \right)^n \left(\frac{3n+1}{n} \right)^n \frac{m_1}{R_o^{3n}} \qquad (6.16)$$

Thus from equations (6.12), (6.14) and (6.15), (6.16) for same flux Q, the non-dimensionalized resistance and the shear stress can be obtained as follows :

$$\bar{\lambda} = \frac{\lambda}{\lambda_n} = \left(\frac{n}{3n+1} \right)^n \frac{R_o^{3n+1}}{B} \int_0^B \frac{dz}{[I(z)]^n} \qquad (6.17)$$

and

$$\bar{\tau} = \frac{\tau_w}{\tau_N} = \frac{1-(dR/dz)^2}{1+(dR/dz)^2} \left(\frac{n}{3n+1} \right) R(z) \left[\frac{R_o^3}{I(z)} \right]^n \qquad (6.18)$$

6.3 Effect of the Peripheral Layer :

To study the effect of the peripheral layer consistency on

$\bar{\lambda}$ and $\bar{\tau}$ we consider the flow to be consisting of a core region with consistency m_1 surrounded by a peripheral layer of consistency m_2. Thus the consistency function $m(r)$, in this case can be written as

$$m(r) = \begin{cases} m_1 & 0 \le r \le R_p(z) \\ m_2 = \beta m_1 & R_p(z) \le r \le R(z) \end{cases} \tag{6.19}$$

where $R_p(z)$ is the radius of central core and peripheral layer has thickness $(R(z) - R_p(z))$.

Following Shukla et al. (1980c) the central layer radius $R_p(z)$ is given by

$$R_p(z) = \alpha R(z) \tag{6.20}$$

where $\alpha < 1$ is a constant.

Using (6.19) and (6.20) and integrating (6.10) we get

$$[I(z)]^n = G(\frac{n}{3n+1})^n R^{3n+1}(z) \tag{6.21}$$

where

$$G = (\frac{1}{\beta}) [1- \{1-\beta^{1/n}\} \alpha^{3+1/n}]^n \tag{6.22}$$

Substituting the expression for $I(z)$ from (6.21) into (6.17) and (6.18) and introducing the following non-dimensionalized variables

$$(\bar{z}, \bar{d}_1, \bar{L}_1, \bar{L}_2, \bar{B}_1) = (z, d_1, L_1, L_2, B_1)/ B,$$

$$\bar{R}(z) = R(z)/ R_o, \bar{R}^*(z) = R^*(z)/ R_o, \bar{\delta}_i = \delta_i/R_o \quad (i = 1,2)$$

we finally get

$$\bar{\lambda} = \frac{1}{G} \int_0^1 \frac{d\bar{z}}{[\bar{R}(\bar{z})]^{3n+1}} \tag{6.23}$$

$$\bar{\tau} = \frac{1 - (d\bar{R}/d\tilde{z})^2 (R_O/B)^2}{1 + (d\bar{R}/d\bar{z})^2 (R_O/B)^2} \cdot \frac{1}{G[\bar{R}(\bar{z})]^{3n}} \tag{6.24}$$

6.4 Results and Discussions :

It can be seen that the results presented in this paper reduce to those of Shukla et al. (1980) when the variable region is abstent and $\beta = 1$. To explicitly see the effects of various parameters, the resistance to the flow and the shear stress acting on the wall have been numerically computed by taking

$$R^*(z)/ R_O = \exp[K(z-B_1)^2] = \exp[\bar{R}(\bar{z}-\bar{B}_1)^2]$$

where $\bar{K} = KB^2$

and

$$\bar{d}_1 = 0.2, \ \bar{L}_1 = 0.2, \ \bar{L}_2 = 0.2, \ \bar{B}_1 = 0.8, \ R_O/B = 0.1, \alpha = 0.95.$$

The effects of heights of stenoses $\bar{\delta}_1$ and $\bar{\delta}_2$, power law index n, wall exponent parameter $\bar{K}$ and consistency ratio β on resistance to the flow $\bar{\lambda}$ are shown in Figs. (6.2)- (6.4). It can be noted from these figures that the resistance increases with increase in heights of stenoses $\bar{\delta}_1$ and $\bar{\delta}_2$ and also with the increase in consistency ratio β. The resistance ratio decreases with decrease in power law index n. It can also be observed that for given values of β and n resistance increases for a convergent tube $(K < 0)$ compared to its values for a straight tube $(K = 0)$ but it decreases for a divergent tube $(K > 0)$. It can also be noted that $\bar{\lambda}$ increases with number of stenoses. This observation is in conformity with the experimental findings of Talukder et al (1976).
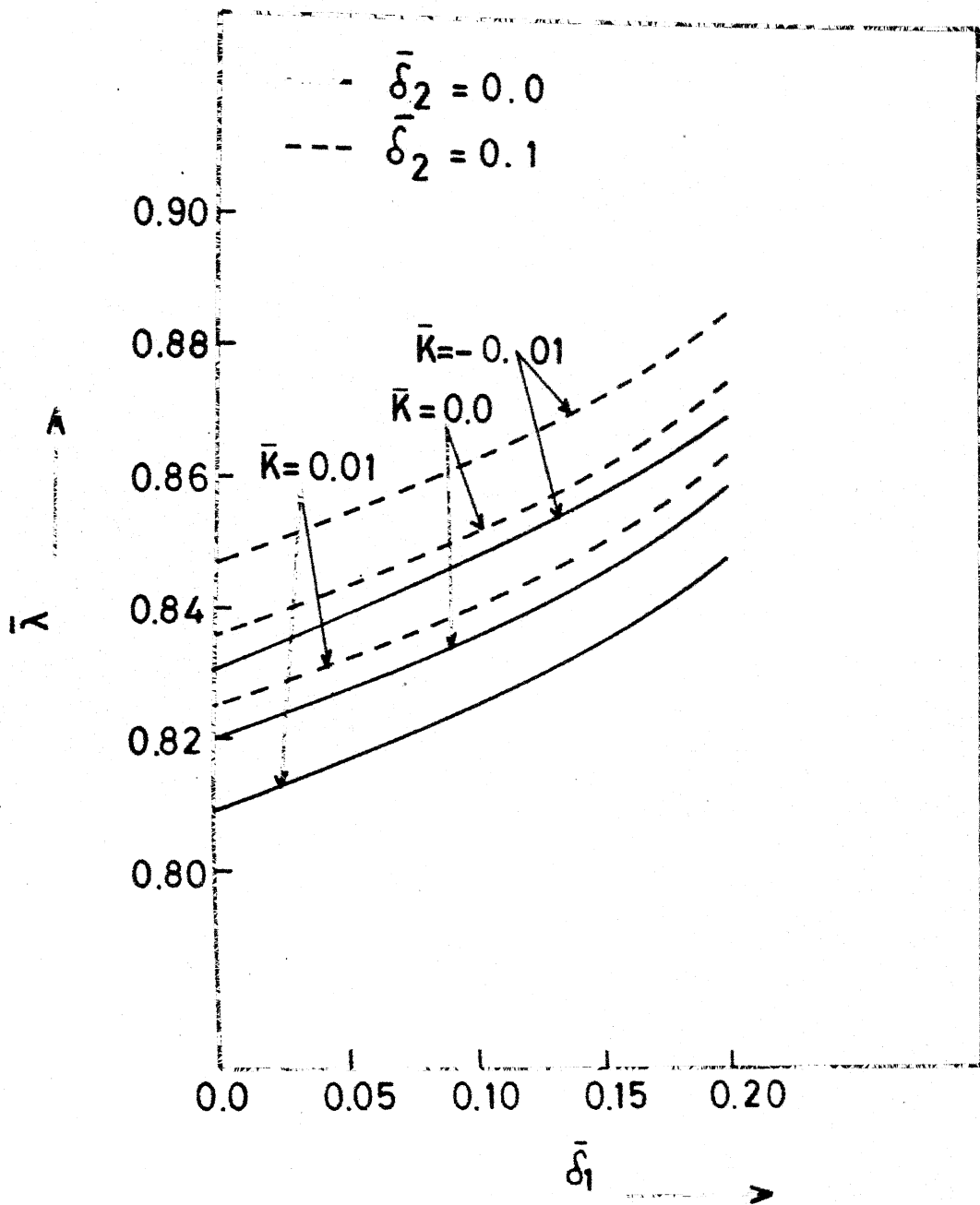
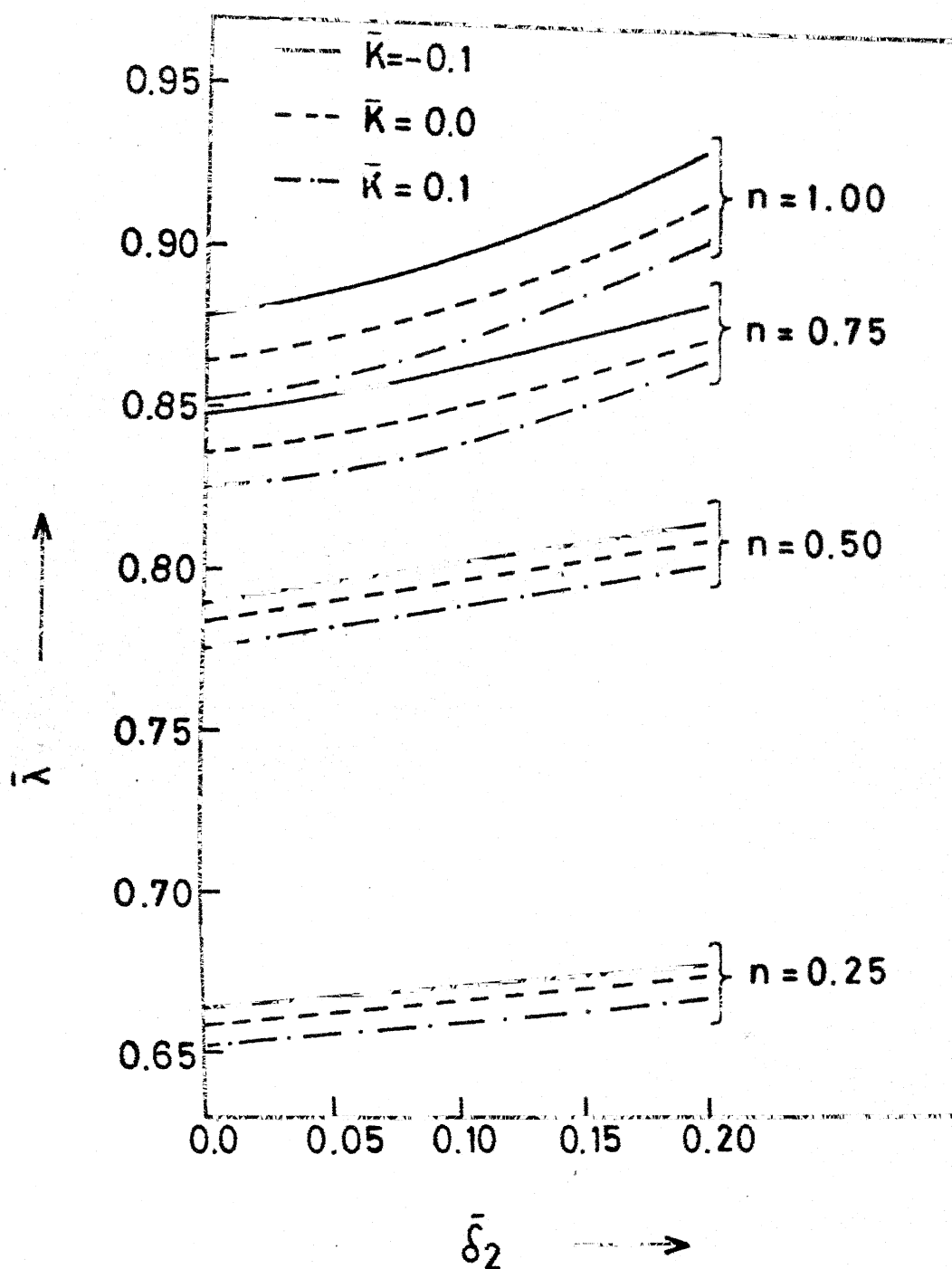Fig.6·2 Effect of $\bar{\delta}_1$ and $\bar{\delta}_2$ on $\bar{\lambda}$ ($n = 0.75$, $\beta = 0.50$)

Fig.63 Effect of $\bar{\delta}_2$ on $\bar{\lambda}$ with variation in n and $\bar{K}$
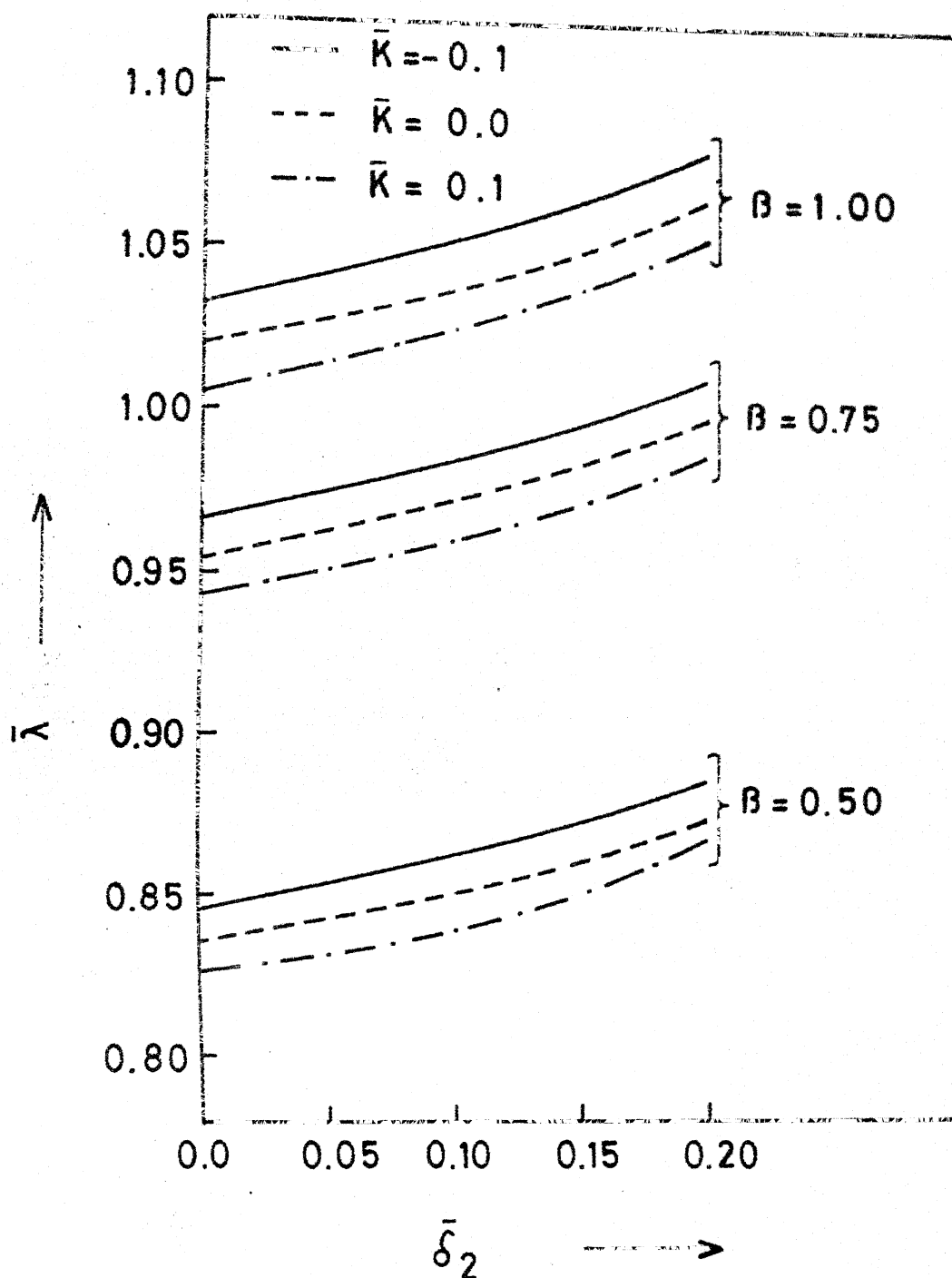($\bar{\delta}_1$ = 0.1 , β = 0.50)

Fig.6·4 Effect of $\bar{\delta}_2$ on $\bar{\lambda}$ with variation in β and $\bar{K}$
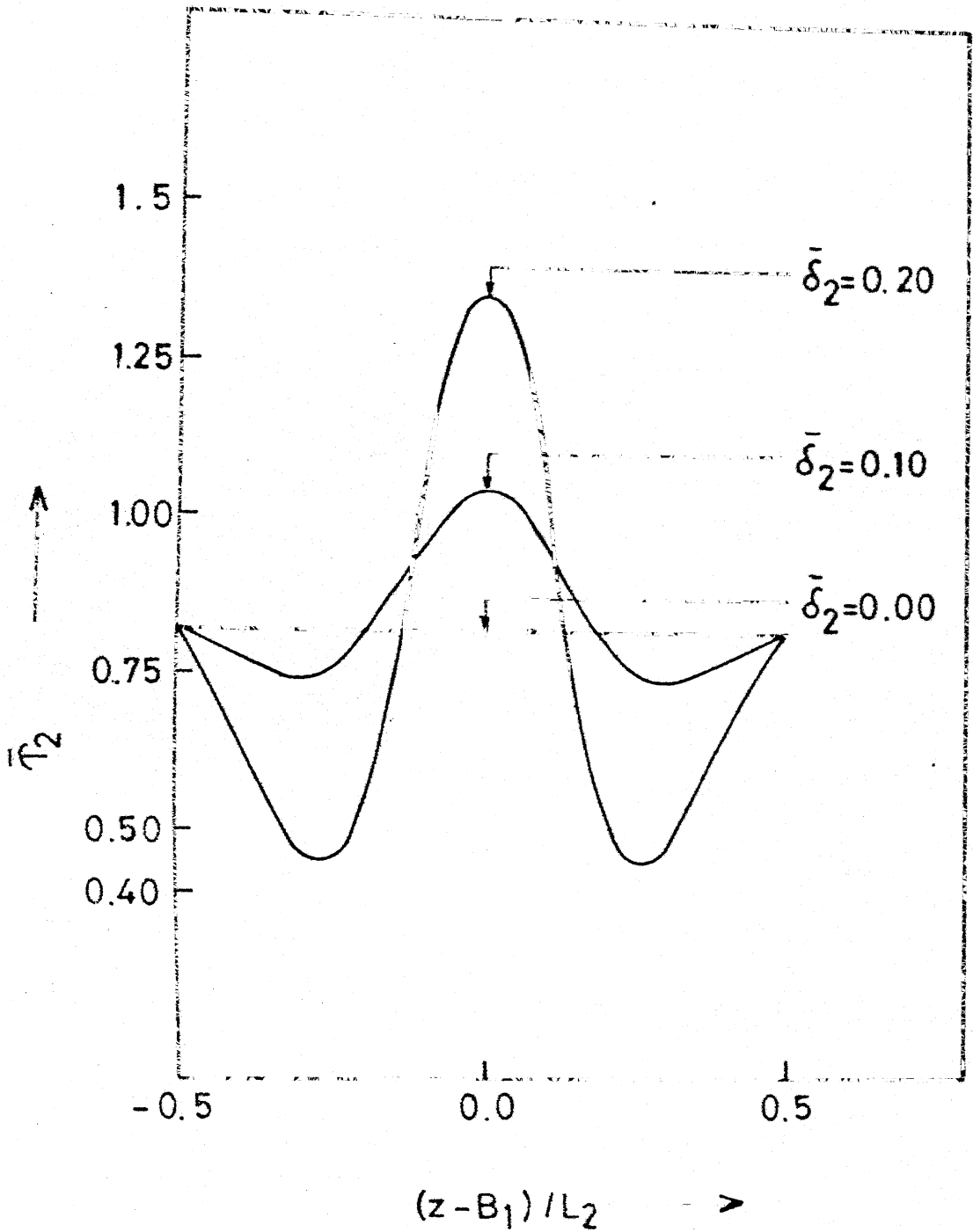$(n = 0.75, \ \bar{\delta}_1 = 0.1)$

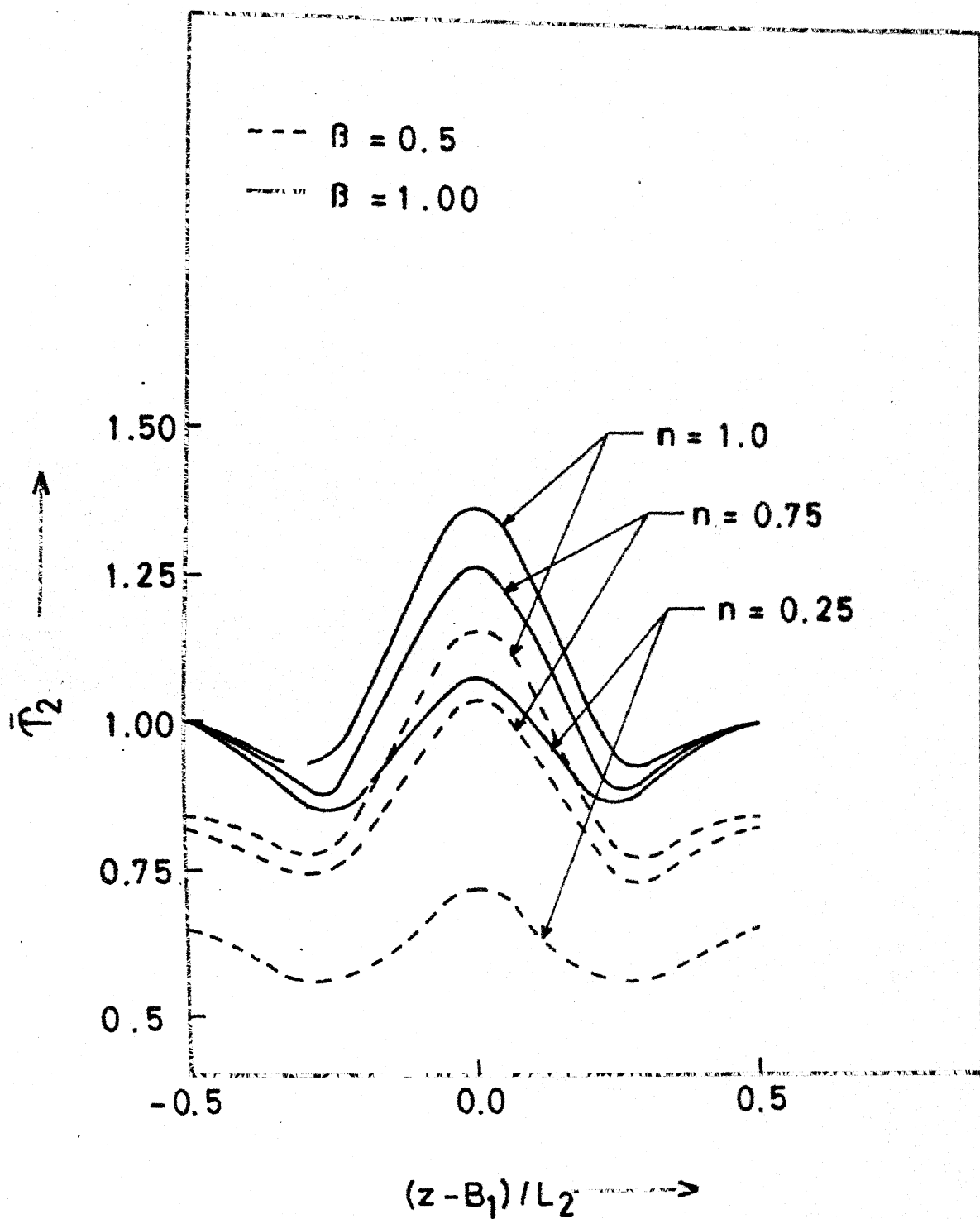Fig. 6.5 Effect of $\bar{\delta}_2$ on $\bar{T}_2$ (n = 0.75 , ß = 0.5)

Fig.6·6 Effect of ß and n on $\bar{T}_2$ ($\bar{\delta}_2 = 0.1$; k = 0)

The shear stress acting over the length of the second stenosis (in the region $|z-B_1| \leq L_2/2$) is shown graphically in Figs.(6.5)-(6.6) for different values of $\bar{\delta}_2$, n and β. It can be observed that the wall shear ratio increases with increase in power law index n. The wall shear also increases with increase in β. It is seen that the value of $\bar{\tau}_2$ oscillates along the length of the stenosis having a maximum value at the mid point of stenosis and two minima at 1/4th and 3/4th of the length of the stenosis. The increase in maximum and decrease in minimum values of $\bar{\tau}_2$ is very significant as $\bar{\delta}_2$ increases.

TABLE 6.1

Effect of $\bar{K}$ on $\bar{\tau}_2$ ($\bar{\delta}_2 = 0.10$, $\beta = 0.5$)

n	$\bar{K}$	$(z-B_1)/L_2$		
		0.1	0.3	0.5
0.25	-0.1	0.6542	0.5604	0.6524
	0.0	0.6541	0.5599	0.6523
	0.1	0.6540	0.5595	0.6521
0.75	-0.1	0.9482	0.7427	0.8205
	0.0	0.9480	0.7420	0.8200
	0.1	0.9479	0.7413	0.8195
1.00	-0.1	1.0473	0.7845	0.8442
	0.0	1.0471	0.7837	0.8435
	0.1	1.0469	0.7829	0.8429

The effect of $\bar{K}$ on $\bar{\tau}_2$ is shown in Table 6.1, for fixed values of β, n and $\bar{\delta}_2$. Since the cross sectional radius of the tube does not vary upto half of the length of the second

stenosis, the parameter $\bar{K}$ does not affect $\bar{\tau}_2$ in that region and thus its values in that region are not shown in table 6.1 It can be noticed that $\bar{\tau}_2$ increases for a convergent tube and decreases for a divergent tube in comparision to its value for a straight tube.

6.5 CONCLUSION :

A matematical model for the steady flow of a non-Newtonian power law fluid with varying cross-section and having two stenoses, one located at the junction, has been presented. Solutions has been obtained for mild stenoses case and it has been shown that the resistance to the flow increases with number of stenoses, for a given flux. This result is in conformity with the experiment by Talukder et al. (1976).

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